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SUMMARY

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This report describes a guidance system, based on terminal control techniques, which was developed to control a capsule-type vehicle from parabolic entry conditions to desired terminal conditions. The guidance system makes use of three separate portions of a precomputed reference trajectory to obtain range control for the vehicle. A skip maneuver is utilized to obtain desired ranges greater than about 4,200 international nautical miles from the initial entry point.

Results of the study are presented which describe the operation of the system for a number of desired terminal conditions and a variety of off-nominal initial entry conditions. Results are also presented which illustrate the operation of the system in nonstandard atmospheres. From a range-error sensitivity standpoint, optimum skip-out conditions are indicated.

The results of the study indicate that the guidance system provides effective range control to desired destinations of from 1,200 to 19,000 nautical miles from the initial entry position.

AUTHOR

INTRODUCTION

The concept of entry guidance about previously computed reference trajectories has been studied extensively. An analysis of this type of range control is given in reference 1 and examples of specific systems are given in references 2 and 3. Because of the necessity of operating in the neighborhood of the reference trajectory these methods are, in general, restricted in range capability to destinations of about 6,000 international nautical miles or less. This limitation could be overcome by using several nominal trajectories but the storage requirements make this procedure undesirable.

One method for extending the range attainable is to guide the vehicle so that after the initial entry a skip maneuver is performed in which the vehicle exits from the atmosphere with conditions allowing a reentry near the desired destination (ref. 4). An attractive feature of this type of entry is the low total-heat input to the vehicle.

In the present analysis the terminal controller technique for reentry guidance used in references 2, 4, and 5 was employed to develop an entry guidance system to control the trajectory of a capsule-type vehicle from parabolic entry conditions to specified terminal conditions. The control system developed uses terminal control techniques to obtain predicted values for certain trajectory variables at specified points during an entry and then uses these predicted values to obtain the desired trajectory control. The control system required storage of three separate portions of a precomputed reference trajectory.

The basic concept of range control studied involves guiding the initial entry to level flight conditions at a predetermined altitude from which point three control procedures are available depending on the desired destination. For minimum range entries, the vehicle remains at a high deceleration level throughout the entry, whereas for longer ranges up to about 4,200 nautical miles a pull-up is made to an intermediate altitude. For desired ranges greater than 4,200 nautical miles a skip maneuver is utilized in guiding the vehicle to the destination. Basically the most difficult of the previously described range procedures is skip-out and a major portion of the present analysis is devoted to this entry maneuver.

In the present analysis the system for selecting skip conditions is derived. An analysis is made to determine the best exit conditions from a range-error sensitivity standpoint. Results of a number of trajectory studies are presented illustrating the operation of the system for a variety of desired terminal conditions. Results are also presented which describe the operation of the system for off-nominal initial entry conditions and for entries into nonstandard atmospheres.

SYMBOLS

a_{ij}	elements in ith row and jth column of matrix of influence coefficients for adjoint solutions
B	constant used in exponential approximation of atmospheric density, ft^{-1}
C_1,C_3,C_4	constants used in control system
C_L	lift coefficient
C_D	drag coefficient
D	drag, lb
g	acceleration due to gravity, ft/sec
G	function used to vary standard atmosphere (eq. (9))

h altitude, ft

K_1, K_2 gain constants used in control equation

L lift, lb

m mass of vehicle, slugs

r_o radius of earth, ft

R_D desired longitudinal range from entry point, international nautical miles

R_b distance traveled by vehicle outside atmosphere during ballistic portion of skip, international nautical miles

R_e range capability during reentry following a skip maneuver, international nautical miles

R_1, R_2 specified ranges used in control logic, international nautical miles

r distance from earth center, ft

S surface area, sq ft

T final or terminal time, sec

t time, sec

V velocity, ft/sec

V_s circular satellite velocity at $h = 300{,}000$ ft, 25,850 ft/sec

V_1 velocity at beginning of phase 3, ft/sec

W weight of vehicle, lb

x dependent variable in differential equation

y dependent variable in adjoint system

$\left.\begin{matrix} y_1, y_2, y_3 \\ y_h, y_\gamma, y_V \end{matrix}\right\}$ influence functions from adjoint system

γ flight-path angle, deg

ϵ specified error in some variable

δ variation of quantity from that of reference trajectory

δ_P	predicted change in variable
η	range traveled by vehicle, deg
ϕ	roll angle, deg
ϕ_r	roll angle as defined by equation (4), deg
ρ	atmospheric density, slugs/cu ft
ρ_c	constant used for approximation of atmospheric density, slugs/cu ft

Subscripts:

D	desired value
e	entry conditions following a skip maneuver
o	initial value
R	reference trajectory value of variable
T	final or terminal time
x	exit conditions for ballistic portion of skip trajectory

A dot over a quantity denotes differentiation with respect to time.

ANALYSIS

The procedures used in developing an entry range control system which utilizes terminal controller techniques are given in the following sections.

Vehicle Characteristics

A capsule-type vehicle with an offset center of gravity was used in the present analysis. This offset center of gravity was such that the vehicle trimmed in the atmosphere at an angle of attack which produced a value of L/D of 0.5. Lift modulation was obtained by varying the roll angle to proportion the lift of the vehicle between the longitudinal and lateral planes. The physical characteristics of the assumed vehicle are given in the following table:

g, ft/sec^2 (average value)	31.2
r_o, ft	2.112×10^7
W, lb	7,000
S, sq ft	106
C_L	0.5
C_D	1.02

Equations of Motion and Assumptions

The force equations solved in the analysis are the following equations of motion:

$$\dot{h} = V \sin \gamma$$

$$\dot{\gamma} = \frac{57.3}{V}\left[\frac{L}{m} \cos \phi - g \cos \gamma + \frac{V^2}{r} \cos \gamma\right] \qquad (1)$$

$$\dot{V} = -\frac{D}{m} - g \sin \gamma$$

$$\dot{\eta} = 57.3 \frac{V}{r} \cos \gamma$$

where

$$L = \frac{C_L S}{2} \rho V^2$$

and

$$D = \frac{C_D}{C_L} L$$

The earth was assumed to be spherical with a radius of 2.112×10^7 feet. A constant gravitational field of 31.2 ft/sec^2 was assumed throughout the altitude range covered. The earth was assumed to be stationary in all respects and there was no relative movement of the atmosphere. An exponential variation of density with altitude was assumed.

Basic Concept

In order to simplify the entry guidance problem, the entry was divided into separate phases. These phases were devised so that, depending upon the desired destination, the control system would select one of three possible procedures for guiding the vehicle to the vicinity of the destination. Control of the final portion of the entry during which the vehicle descended to the desired destination was not considered in the present study as it has been previously covered rather thoroughly in the literature (refs. 2 and 3). Thus, the objective of the present control system was to safely guide the vehicle from parabolic entry conditions to a point from which a final descent could be made to the desired destination.

Lateral-range control was not considered in the present study since previous studies (ref. 3) have shown that effective lateral-range control can be achieved by utilizing available lift in the lateral plane to guide the vehicle on to the correct heading.

A general description of the separate phases is given with the aid of figure 1. Figure 1 shows a profile plot of altitude and range for each phase of an entry.

In phase 1 the vehicle initial entry trajectory was controlled so that the vehicle arrived at an altitude of about 215,000 feet at which time its flight-path angle was near zero. The vehicle then proceeded into phase 2 of the entry which consisted of maintaining a constant altitude as shown in figure 1. If the desired destination was less than R_1 (2,000 international nautical miles) from the reentry point, the control remained in phase 2 until the descent reference trajectory was attained. If the desired destination was between R_1 and R_2 (2,000 and 4,200 nautical miles), the vehicle continued in phase 2 until the velocity was reduced to some predetermined value V_1 (29,000 ft/sec) and then entered into phase 3 as shown in figure 1. Phase 3 consisted of a pull-up to an intermediate altitude of about 250,000 feet followed by a coasting period at this altitude to the desired destination. For ranges greater than R_2 (4,200 nautical miles) the vehicle passed from phase 2 into phase 4 which involved controlling the trajectory such that a skip-out occurred. During phase 4 the trajectory was controlled so that the exit conditions were such that the vehicle would reenter near the desired destination at which time a final descent to the landing area could begin.

Thus, for desired ranges less than 4,200 nautical miles from the initial entry point a direct entry was made in which the vehicle remained in the atmosphere throughout the entry while for ranges greater than 4,200 miles a skip maneuver was employed. This 4,200-nautical-mile switching point was chosen to allow a sufficient range capability overlap between the direct and skip entry guidance procedures as will be shown in a subsequent section of the paper. This is also the approximate range at which a trade-off occurs between direct and skip entries with respect to the total aerodynamic heat input (ref. 3).

The manner in which terminal control techniques were used in guiding the vehicle during each of the previously described phases will be discussed in following sections of the paper. A brief description of the terminal control technique along with its application to a typical problem is given in the appendix.

REFERENCE TRAJECTORIES AND GUIDANCE PROCEDURES

The initial objective of this study involved the selection of suitable reference trajectories for the separate phases of the entry. Adjoint solutions were then computed for these reference trajectories in order to obtain influence functions to be used in predicting terminal values of certain trajectory variables. These predicted values were then used to obtain the desired trajectory control.

The reference trajectories, influence function, prediction and control equations, and control logic used during each phase of the entry are now presented. The application of these procedures to the control of a variety of simulated entries is discussed in the section "Discussion of Results."

Phase 1

The reference trajectory for the initial atmospheric entry phase of the mission is shown in figure 2. The initial conditions for this trajectory were an altitude of 400,000 feet, a velocity of 36,000 ft/sec, and a flight-path angle of -5.3°; the final conditions were an altitude of about 215,500 feet and a flight-path angle of 0°. This trajectory was achieved by maintaining a constant roll angle of 90° (Lift = 0 in longitudinal plane) throughout the trajectory as is shown in figure 2. A zero-lift trajectory was chosen so that maximum longitudinal control of the vehicle trajectory would be available about the reference trajectory. This trajectory also produced a maximum heading change during phase 1 since all the lift of the vehicle could be directed into the lateral plane.

Trajectory control was achieved in phase 1 by predicting the terminal error in altitude (at $h = 215{,}500$ ft) and by controlling the vehicle lift so that this terminal error approaches zero. The influence functions used in this prediction are shown in figure 3. The influence functions are normally represented as functions of time (appendix). However, in the present analysis the influence functions were given as functions of altitude as shown in figure 3. By using altitude as the independent variable certain simplifications could be achieved. Since the error in altitude (δh) was always zero the storage of influence functions was reduced from three to two. (See eq. (A4).) Since time was not used directly, problems arising from the fact that the entry time varied for different initial conditions were eliminated.

Although the altitude influence function was not used in the prediction scheme, it was still necessary to compute it since the computation of y_{γ} and y_V required the use of y_h. The initial conditions for the adjoint equations can be determined, as was done in the example in the appendix, to be given by

$$y_h(T) = 1$$

$$y_{\gamma}(T) = 0$$

$$y_V(T) = 0$$

The prediction equation for phase 1 is given by

$$\delta_P h_T = \left(h_{R,T} - h_D\right) + y_{\gamma}(h)\delta\gamma + y_V(h)\delta V \qquad (2)$$

The first term on the right-hand side of equation (2) was always zero in the present study since the desired altitude coincided with the final reference altitude.

A close examination of equation (2) and figure 3 gives some insight into the physical interpretation of the influence functions and the prediction equation. For example, consider an entry which at an altitude of 300,000 feet has a velocity 200 ft/sec less than the reference velocity (δV = -200 ft/sec) and a flight-path angle 0.1° steeper than the reference flight-path angle ($\delta\gamma = 0.1^\circ$). At this altitude the value for y_V is about $6 \frac{\text{ft}}{\text{ft/sec}}$ (fig. 3). Hence the predicted error in the final altitude due to the velocity perturbation is -1,200 feet. The value for y_γ at this altitude is 52,000 ft/deg and hence the predicted error in final altitude due to $\delta\gamma$ is 5,200 feet. Thus, the combination of δV and $\delta\gamma$ gives a predicted final error in altitude of 4,000 feet.

A simple off-on control system was used to reduce the terminal altitude error given by equation (1). The control logic scheme used in determining the orientation of the lift vector of the vehicle is given by the following relations:

$$\delta_p h_T > C_1 \qquad (\phi = 180^\circ)$$

$$\delta_p h_T < -C_1 \qquad (\phi = 0)$$

$$-C_1 < \delta_p h_T < C_1 \qquad (\phi = 90^\circ \text{ (reference value)})$$

Thus, maximum lift was used in bringing the altitude error inside a specified deadband C_1.

Phase 2

During this phase of the mission the vehicle maintained a constant altitude of 215,500 feet. This altitude was achieved by regulating the vehicle lift such as to effect a balance between the lift, centrifugal force, and gravity force acting on the vehicle. The control law used in accomplishing this requirement is given by

$$\phi = \phi_r + K_1\left(h - h_D\right) + K_2\dot{h} \tag{3}$$

where ϕ_r is computed from the equation for $\dot{\gamma}$ (eq. (1)) by setting $\dot{\gamma}$ and γ equal to zero and is given by

$$\phi_r = 57.3 \cos^{-1}\left[\frac{W\left(1 - \frac{V^2}{gr}\right)}{L}\right] \tag{4}$$

Thus, by regulating the vehicle roll angle in accordance with equation (3) the constant altitude desired in phase 2 could be maintained.

Phase 3

Phase 3 of the mission consisted of a pull-up, initiated at a specified velocity, to an altitude of about 250,000 feet from where a coasting period to the desired destination could begin. The reference trajectories and influence functions for this phase of the entry are shown in figures 4 and 5.

The prediction equation and control logic for phase 3 of the mission are the same form as those for phase 1, the only difference being the values for the reference quantities, constants, and influence functions.

Phase 4

The reference trajectory for the skip phase of the entry was computed such that the vehicle exited from the atmosphere $\left(h_x = 300{,}000 \text{ feet}\right)$ with a velocity near satellite velocity and a flight-path angle of about 2°. These exit conditions were found to be near optimum from a range-error sensitivity standpoint as is shown in a subsequent section of the paper. The phase 4 reference trajectory is shown in figure 6.

Since the range traveled outside the atmosphere following a skip-out is a function of the exit velocity and flight-path angle, these quantities were predicted during the skip maneuver. Thus influence functions had to be precomputed for both the velocity-prediction equation and the flight-path-angle-prediction equation. These influence functions are shown in figure 7. The prediction equations have the same form as before and are given by

$$\left.\begin{aligned} \delta_P V_x &= \left(V_{R,x} - V_{D,x}\right) + y_\gamma(h)\delta\gamma + y_V(h)\delta V \\ \delta_P \gamma_x &= \left(\gamma_{R,x} - \gamma_{D,x}\right) + y_\gamma(h)\delta\gamma + y_V(h)\delta V \end{aligned}\right\} \tag{5}$$

where y, of course, has a different value in each equation resulting from different initial conditions.

The solution of equations (5) requires exit values for $V_{D,x}$ and $\gamma_{D,x}$. In the simulation the value for the desired flight-path angle $\gamma_{D,x}$ was specified for each entry whereas the desired exit velocity was computed from orbital considerations as follows. By specifying an exit flight-path angle and a desired ballistic range outside the atmosphere R_b the velocity required for obtaining this range is given by the following equation which is from reference 6 and given in the notation of the present paper:

$$V_{D,x} = \frac{V_{s,x}}{\cos \gamma_{D,x}} \left[\frac{\tan(R_b/120)}{\tan(R_b/120) + \tan \gamma_{D,x}}\right]^{1/2} \tag{6}$$

Solutions to equation (6) for a variety of exit conditions are shown in figure 8 and are also given in figure 3 of reference 6.

The control equation for phase 4 was devised by considering the error in range at skip-out due to the predicted errors in exit velocity and flight-path angle. This error in range is given by the following equation:

$$dR_b = \frac{\partial R_b}{\partial \gamma_x} \delta_P \gamma_x + \frac{\partial R_b}{\partial V_x} \delta_P V_x \tag{7}$$

The partial derivative terms in equation (7) can be determined from equation (6) and are given by

$$\left.\begin{aligned} \frac{\partial R_b}{\partial \gamma_x} &= -120\left(\frac{V_x}{V_{s,x}}\right)^2 \left\{\frac{1 - \left[2 - \left(V_x/V_{s,x}\right)^2\right]\cos^2\gamma_x}{1 - \left(V_x/V_{s,x}\right)^2\left[2 - \left(V_x/V_{s,x}\right)^2\right]\cos^2\gamma_x}\right\} \\ \frac{\partial R_b}{\partial V_x} &= \left(13.752 \times 10^3\right)\left(\frac{V_x}{V_{s,x}^2}\right)\left\{\frac{\sin \gamma_x \cos \gamma_x}{1 - \left(V_x/V_{s,x}\right)^2\left[2 - \left(V_x/V_{s,x}\right)^2\right]\cos^2\gamma_x}\right\} \end{aligned}\right\} \tag{8}$$

where V_x and γ_x are given by

$$V_x = V_{D,x} + \delta_P V_x$$

$$\gamma_x = \gamma_{D,x} + \delta_P \gamma_x$$

Solutions to equations (8) are given in figure 9 and in figure 4 of reference 6.

Thus, a solution of equation (7) gives a predicted value for the range error at exit. Although the exit velocity and flight-path angle may differ somewhat from the desired values at exit, the combination of these exit errors should be such that the range error at exit is zero.

The control logic used in reducing the range error at exit is given by

$$\left.\begin{array}{lr} dR_b > C_3 & (\phi = 180^\circ) \\ dR_b < -C_3 & (\phi = 0) \\ -C_3 < dR_b < C_3 & (\phi = 90^\circ) \end{array}\right\} \text{for} \quad V_x < V_{s,x}$$

$$\left.\begin{array}{lr} dR_b > C_3 & (\phi = 0) \\ dR_b < -C_3 & (\phi = 180^\circ) \\ -C_3 < dR_b < C_3 & (\phi = 90^\circ) \end{array}\right\} \text{for} \quad V_x > V_{s,x}$$

The significance of this logic reversal for exit velocities above satellite velocity is discussed in a later section of the paper.

In order to effect a smooth transition between the level flight phase and the skip phase of the entry, an additional prediction was used to determine the desired pull-up velocity. During the constant-altitude phase a continuous prediction of the exit velocity error was made using the initial phase 4 reference values (at $h = 215{,}000$ feet) for velocity, flight-path angle, and the influence functions. This velocity-prediction equation was the same as is given in equations (5). Thus, the exit velocity error was continuously computed during phase 2 under the assumption that the skip maneuver was initiated at the point that the computation was made. When this velocity error $\delta_p V_x$ was reduced below a predetermined value C_4 the trajectory control passed from the constant-altitude phase to the skip phase.

Although the final objective of the previously described guidance system was to control entry range, no range influence function was required. This simplification was made possible by dividing the entry into separate phases such that during any particular phase the quantities predicted were not functions of range.

Effect of Exit Conditions on Range Capability During Reentry Following a Skip Maneuver

The exit conditions for the previously described phase 4, skip portion of a trajectory, were chosen as a result of the following analysis.

The range capability during reentry following a skip maneuver is shown in figure 10. For the ideal case shown in figure 10, this range capability is bounded by the trajectories for $L/D = 0$ and $L/D = 0.5$. This range capability is, of course, a function of the exit conditions for the skip $\left(V_e = V_x, \quad \gamma_e = -\gamma_x\right)$ and is about 1,600 nautical miles for a skip angle of 2° at satellite velocity.

In an actual entry, the conditions at exit would be in error due to instrument inaccuracies and errors in the inertial guidance system. (See ref. 7.) Thus, the ideal range capability shown in figure 10 would be reduced by these uncertainties in the exit conditions. For example, if the desired destination were in the center of the boundaries shown in figure 10, the vehicle would have a certain maneuver capability about this desired destination. If errors existed at exit, these errors would change the distance traveled outside the atmosphere and hence at entry the maneuver capability would be shifted to the right or to the left with respect to the desired destination, with a reduction in the range capability resulting.

In order to study this effect, calculations were made of the reentry range capability for different assumed errors in exit velocity and flight-path angle. Figures 8 and 9 were used in computing the effect on the ballistic range for these errors. Figure 11 shows the entry range capability for different entry angles, skip ranges, and assumed errors.

The data of figure 11 were obtained by adding the magnitude of the separate range errors due to velocity and flight-path angle. This was a conservative estimate since the errors might cancel each other.

Figure 11 shows that as the assumed errors in $\epsilon_{V,x}$ and $\epsilon_{\gamma,x}$ were increased the range capability was decreased. Also, for small errors in the exit conditions $\left(\epsilon_{V,x} = 3.3 \text{ ft/sec}, \quad \epsilon_{\gamma,x} = 0,02^{\circ}\right)$ the range capability increased with decreasing entry angle. However, for larger errors the maximum range capability occurred for entry angles between -1° and -2°. This peak results from the large effect of exit errors at small flight-path angles as shown in figure 9.

ATMOSPHERIC DENSITY VARIATIONS

In order to study possible variations in density about the assumed standard atmosphere, the density was represented by the following relationship:

$$\rho = G(h) \times \text{Standard density} \tag{9}$$

where the standard density was given by $\rho_R = \rho_c e^{-Bh}$, and $G(h)$ was represented by a table so that the standard atmosphere could be varied as a function of altitude to represent a nonstandard atmosphere such as might be experienced during a polar entry.

Since all reference trajectories were computed by using the standard atmosphere and since equal density levels occur at different altitudes for nonstandard as compared with standard atmospheres, it was desirable to relate the altitudes traversed in a nonstandard atmosphere to those traversed in the standard atmosphere. This relation was accomplished by using density altitude rather than geometric altitude in determining reference values for velocity and flight-path angle used in the control equations. Thus, by knowing the atmospheric density at a specific altitude during an entry into a nonstandard atmosphere, the altitude corresponding to that density for the standard atmosphere can be determined by equating the density equations for each atmosphere to get

$$h_R = \frac{1}{B} \log_e \frac{\rho_c}{\rho} \tag{10}$$

By substituting the density ρ into equation (10) the corresponding reference altitude could be determined and hence reference values for velocity, flight-path angle, and the influence functions could be determined.

In an actual application the influence functions and reference values for velocity and flight-path angle could be stored as some function of a reference atmospheric density. During an entry the actual atmospheric density could be determined (from deceleration measurements) and used in specifying the reference values for V, γ, and the influence functions; thus, altitude need not be determined during the entry. This is an attractive feature since it is well known that an instability exists in the determination of altitude from an inertial platform. (This error in the computation of altitude would probably not be a factor for the present system (ref. 7) because of the short times involved in performing the entry maneuver.)

Two arbitrary types of variations from the standard atmosphere were considered and are illustrated in figure 12. One type of variation was obtained by multiplying the standard density by a constant factor (G = Constant in eq. (9)). This variation could result from a fixed change in the value of the atmospheric pressure at sea level ρ_c and is illustrated in figure 12 by the solid line representing a 100-percent change in ρ_c. A second type of change was considered in which the atmosphere varied in some fashion with altitude and is illustrated in figure 12 by the dashed line for which the density increased linearly from the nominal value at 400,000 feet to 100 percent of the nominal at 200,000 feet. This variation in the gradient of the atmosphere might result from changes in ρ_c with range traveled or from a change in the gradient of the atmosphere B with altitude.

DISCUSSION OF RESULTS

Direct Entries

An example of a typical entry during which the vehicle remained in the atmosphere is shown in figure 13. The initial conditions for this entry were the same as for the phase 1 reference trajectory, and a range between 2,000 and 4,200 nautical miles was desired.

As is shown by figure 13 the roll angle in phase 1 remained at the nominal value since the predicted error in the pull-out altitude was less than an assumed 1,000-foot deadband about the desired pull-out altitude. This 1,000-foot deadband about the desired altitude in phase 1 was used throughout the study to prevent unnecessary control changes which would occur in attempting to attain a precise final altitude condition.

During phase 2 of the entry the vehicle maintained a constant altitude until the velocity was reduced to 29,000 ft/sec. The gains used in the control equation (eq. (3)) during phase 2 were $K_1 = 0.005^\circ/\text{ft}$ and $K_2 = 0.075^\circ/\text{ft/sec}$. These gains were found to be adequate for establishing and maintaining level flight.

After establishing the desired velocity in phase 2 the entry proceeded into phase 3 as shown in figure 13. The control logic reduced the roll angle to zero in order to establish a rate of climb. As the predicted error in final altitude approached zero, the roll angle went to 180° causing the vehicle to level out at an altitude of about 255,000 feet. This slight overshoot in the desired altitude is of little significance since sufficient lift is available below 260,000 feet to maintain level flight at satellite velocity.

The time history shown in figure 13 was terminated once a level flight condition was achieved. At this point ($t = 284$ seconds) the vehicle had traversed a range of 1,500 nautical miles and was traveling at near satellite velocity. From these conditions it would be an easy task to control the vehicle to a reference trajectory terminating at a point between 2,000 and 4,200 nautical miles from the initial entry point (ref. 3).

If a desired range less than 2,000 nautical miles had been desired, the vehicle would have remained in phase 2 until the final reference trajectory was reached. Ranges between about 1,200 and 2,000 nautical miles could be attained by using this procedure (ref. 3).

Skip Entries

For desired ranges greater than about 4,200 nautical miles it is necessary for the vehicle to climb out of the dense atmosphere in order to conserve sufficient energy to reach the desired destination. In the present analysis this was achieved by executing a skip maneuver so that a large portion of the desired range was traversed outside the atmosphere. The manner in which these entries were accomplished by using terminal control techniques is now given. Unless specified otherwise the standard atmospheric density was assumed.

Skip entries with $V_x < V_{s,x}$.- Presented in figure 14 is a time history for a skip entry with a desired range of 10,000 nautical miles and an exit velocity below satellite velocity. Included in this desired range is the distance traveled from the initial entry conditions to the skip-out point plus the ballistic range traveled from the skip-out altitude to reentry at the same altitude. The range traveled during the second reentry was not included since range control during this phase of the entry was not considered.

The initial entry conditions for figure 14 were the same as those for the phase 1 reference trajectory so that during this phase of the entry the roll angle remained at the reference value as shown in this figure. As the entry entered into phase 2 the vehicle followed a constant altitude trajectory and the guidance equations predicted the desired pull-up velocity. As $\delta_p V_x$ approached zero, the vehicle entered into phase 4 and maximum lift was called for to reduce the error in the predicted range at skip-out dR_b. As shown in figure 14, when the predicted errors in exit velocity and flight-path angle have values which caused the range error to approach zero, the vertical lift was reduced to zero ($\phi = 90^\circ$) and remained at this value to the skip-out point. For this entry as for all skip entries the desired exit angle was 2° and for control purposes a 100-mile deadband about $dR_b = 0$ was assumed ($C_3 = 100$).

From figure 14 it is seen that at the beginning of phase 4 the predicted range error at exit dR_b was negative. This condition was achieved by delaying the pull-up until $\delta_p V_x$ was slightly negative and was necessary for a successful skip maneuver with $V_x < V_{s,x}$ as explained in the following discussion.

Because of the nature of the guidance system, range errors could most effectively be corrected during a skip maneuver by extending the lifting period of the vehicle over that used for the reference trajectory. In order to establish a condition at pull-up so that extended lift could reduce range errors it was desirable to initially have a negative value for $\delta_p V_x$ (causing a corresponding negative $\delta_p \gamma_x$) and hence for dR_b. An examination of figure 8 will illustrate this point. As long as the predicted exit conditions were such that dR_b was negative (to the left of a desired constant range line in fig. 8), this range error could be easily corrected by extending the lifting period since this would cause an increase in the flight-path angle (over that of the reference trajectory) and a corresponding increase in velocity because the vehicle would be ascending in the atmosphere more rapidly and hence the drag would be decreasing more rapidly than for the reference case. Thus, errors in velocity and flight-path angle and hence range could be decreased simultaneously. Suppose that the control logic were such that a pull-up were begun with an initial positive range error. To remove this error either δV_x, $\delta \gamma_x$, or both would have to be reduced during the pull-up. Thus, it would be necessary to invert the lift vector ($\phi = 180^\circ$) at some point during the pull-up which would be an undesirable procedure.

The previous discussion was valid only for exit velocities less than satellite velocity as is shown in the next section of the paper.

As is shown in figure 8, skip ranges of up to about 11,000 nautical miles can be achieved for exit velocities below satellite velocity. Numerous entries were computed with desired ranges R_b between this upper limit of 11,000 nautical miles and a lower limit of 2,000 nautical miles. It was found that control of the trajectory to the desired terminal conditions could be achieved for all ranges between the previously mentioned limits. As was described in the previous section, it was possible to make direct entries with ranges as small as 1,200 nautical miles. Thus, the minimum range attainable by using a skip maneuver would be this distance (1,200 nautical miles) plus the minimum distance traveled during the skip portion (2,000 nautical miles) plus the distance traveled during the atmospheric pull-up portion (about 600 nautical miles). This total of about 3,800 nautical miles is well below the maximum range attainable during direct entry (4,200 nautical miles); therefore, an adequate "overlap" in range capability is available between the two entry procedures.

Skip entries with $V_x > V_{s,x}$.- Presented in figure 15 is a time history for a skip entry with a desired range of about 16,500 nautical miles and an exit velocity greater than satellite velocity. Phase 1 and the initial portion of phase 2 for this entry are identical to those described in figure 14. However, the pull-up was initiated slightly earlier (positive $\delta_p V_x$) for this entry causing an initial positive value for dR_b. This was a desirable pull-up condition as can be shown by referring to figure 8. For exit velocities greater than satellite velocity the slope of constant range lines changes signs. Thus, in order for the vehicle to be in a condition so that increased lift can correct range errors at skip-out, the pull-up conditions must be such that the initial range error is positive (to the right of the desired constant range curve in fig. 8) since increased lift increases γ and hence decreases R_b. This explains the necessity for the previously discussed control logic reversal for exit velocities greater than satellite velocity.

Successful entries were computed for desired ranges up to about 17,500 nautical miles (R_b = 16,000 nautical miles). This upper range limit was determined by the exit angle chosen and by the requirement that the altitude of the vehicle should not exceed 400 miles during the skip portion of the trajectory. (See fig. 8.)

The total range capability of the vehicle is larger than the previously given figures since the vehicle travels an appreciable distance during the reentry (about 1,500 nautical miles (fig. 11)). Thus, the maximum range attainable by the vehicle would be about 19,000 nautical miles.

Variations in Initial Entry Velocity

In order to study the effects of variations in the initial velocity, entries were calculated for initial velocities less than and greater than the reference value of 36,000 ft/sec. Some results of this study are shown in figure 16.

Presented in this figure is phase 1 of entries for which the initial velocity was either 2,000 ft/sec greater than or less than the reference value. Only phase 1 is shown in figure 16 since once the conditions ($h = h_D$, $\gamma = 0$) are met for the beginning of the constant-altitude phase the remaining phases of the entry are similar to those shown in previous figures. Thus, if the vehicle can be guided to the desired constant-altitude conditions then range control to the desired destination can be achieved. The only effect on range capability would be a small shift in the total range capability of the vehicle. For example, if the initial velocity were greater than the reference value, the vehicle would remain in the constant-altitude phase for a longer period and hence the minimum and maximum ranges obtainable would be slightly greater than those given previously.

Figure 16 shows that, for the entry with an initial velocity 2,000 ft/sec greater than the reference velocity, the control system initially predicted that the vehicle would miss the desired perigee altitude by about 25,000 feet (25,000 feet greater than the desired altitude). This resulted from the increased centrifugal force tending to decrease the flight-path angle. To overcome this effect, the control logic initially selected a roll angle such that maximum negative lift was used for the first 80 seconds of the entry. At this point the predicted error in altitude was reduced to within an assumed 1,000-foot deadband about the desired perigee altitude and the roll angle was reduced to the reference value. Thus, the vehicle arrived at the desired perigee altitude with a zero flight-path angle as is shown in figure 16.

For the entry with an initial velocity less than the reference value, maximum positive lift was used throughout much of the entry to avoid falling below the desired final altitude.

Numerous entries were calculated for a variety of initial velocities. The results have shown that excellent control of the vehicle's trajectory to the desired altitude in phase 1 could be achieved for velocities up to 2,500 ft/sec greater than or less than the reference value of 36,000 ft/sec.

Variations in Initial Entry Angle

The present analysis included a study of the effects of varying the initial entry angle from the reference value of -5.3°. Some results of this study are presented in figure 17. This figure shows time histories of phase 1 for entries with an initial flight-path angle of $1/2^{\circ}$ greater than and $1/2^{\circ}$ less than the reference value. Only phase 1 is shown for the same reasons as were given in the previous section. Figure 17 shows that the initial predicted errors in perigee altitude were successfully corrected and the vehicle arrived at the desired terminal conditions. Note the difference in the times taken for phase 1 of the entries shown in figure 17. This difference illustrates the advantage of using altitude rather than time as the independent variable in storing the reference quantities.

Trajectories were computed for a variety of initial entry angles and it was found that control to the desired terminal conditions could be achieved over the entire range capability of the vehicle for off-nominal conditions of as much as 0.8° greater than or less than the reference value.

While lateral range has not been considered, an examination of figures 13 to 17 yields an important fact regarding the lateral range capability associated with the previously described guidance procedures. During the high deceleration phase of the entry the lift is largely directed in the lateral plane ($\phi \approx 90^{\circ}$). Thus, large heading changes could be achieved early in the entry resulting in near maximum lateral ranges.

Variations in Atmospheric Density

The study included an investigation of the effects of changing the atmospheric density from the assumed variation used in computing reference trajectories and influence coefficients.

Effect of changes in the atmospheric density at sea level.- Presented in figure 18 is a time history of altitude for a skip entry for which the atmospheric density was assumed to be twice that of the reference density throughout the altitude range covered. Shown in this figure is the actual altitude profile along with the corresponding altitude profile from which the reference values were obtained. In other words, the solid line in figure 18 is the actual path which the vehicle followed, whereas the dashed line is the path which the control system thought the vehicle was following. The initial conditions for this entry were the same as for the one shown in figure 14. The initial altitude was, of course, greater than that in figure 14 since an upward shift in altitude was necessary in order to maintain the initial atmospheric density at the same value as for the reference trajectory. (See eqs. (9) and (10).)

An examination of figure 18 shows that this initial altitude increment of about 16,000 feet was maintained throughout the entry. This was, in fact, the only difference in the entries shown in figures 18 and 14 as all other trajectory variables followed the same variations. This result was found to be true in all entries for which the reference density was altered by a constant factor throughout the assumed altitude range. Thus, control of the vehicle to the desired terminal conditions could be achieved for any assumed atmosphere resulting from off-nominal values of the density at sea level. These variations merely resulted in an upward or downward shift in the vehicle's altitude profile, depending upon whether the density was greater than or less than the standard density.

Effect of changes in the gradient of the atmosphere.- Presented in figure 19 is a time history of an entry for which the standard atmosphere was varied in the manner shown by the dashed line in figure 12. The initial conditions for this entry were the same as for the phase 1 reference trajectory and the desired range was 10,000 nautical miles.

An examination of figure 19 shows that during phase 1 of the entry the predicted perigee miss distance gradually increased for the first 40 seconds. This increase resulted because the vehicle was descending into a denser atmosphere than the reference atmosphere and hence at equal density altitudes the flight-path angle was steeper for the entry shown in figure 19 than for the reference trajectory. To overcome this effect, the control system called for maximum upward lift and, inasmuch as this control became more effective as the vehicle descended into denser atmosphere, the predicted altitude error was reduced to zero as is shown in the figure. A similar effect in the predicted errors at exit can be observed in figure 19 during the skip-out phase of the mission.

During phase 4 of the entry more than one control change was required to maintain the range error within the desired limits. For all entries studied other than those with variations in the atmospheric gradient only one control change was required during any phase where terminal control techniques were used. This apparent minimum number of required control changes was pointed out in reference 2.

Numerous entries were computed by using different assumed variations in the gradient of the atmosphere. The results of these calculations have shown that the control system could adequately compensate for changes in ρ_c with altitude of up to $2.1 \times 10^{-8} \frac{\text{slug/cu ft}}{\text{ft}}$ of altitude and changes in the decay factor B of up to 0.22×10^{-10} ft^{-1}/ft. This change in ρ_c amounts to slightly over a 100-percent change per 200,000 feet of altitude. Since in actual practice ρ_c would most likely change with range, this might be thought of as a 100-percent change in ρ_c per 600 nautical miles downrange $\left(0.7 \times 10^{-5} \frac{\text{slug/cu ft}}{\text{nautical mile}}\right)$ since this is the approximate distance the vehicle traveled during either the entry or pull-up phases of the mission. The allowable change in the decay factor B can be interpreted as a 10-percent change in B per 200,000 feet of altitude. Although this 10-percent change in B appears small, it should be noted that for the case illustrated in figure 19 a 10-percent change in B at an altitude of 200,000 feet had the same effect on density as a 100-percent change in ρ_c would have at the same altitude.

SUMMARY OF RESULTS

The results of a study of an entry guidance system which utilizes terminal control techniques to control the trajectory of a capsule-type vehicle from parabolic entry conditions to a desired destination can be summarized as follows:

1. The guidance system required computer storage of three portions of a precomputed reference trajectory. By storing these reference quantities as a function of density altitude the guidance system was able to compensate for a variety of off-nominal entry conditions.

2. The guidance system provided trajectory control to desired destinations between 1,200 and 19,000 international nautical miles from the initial entry point. For desired ranges greater than about 4,200 nautical miles a skip maneuver was used.

3. The guidance system worked effectively for initial off-nominal variations in velocity of up to $\pm 2{,}500$ ft/sec and for initial entry angles as much as 0.8° greater than or less than the reference value.

4. The control system was able to compensate for the following variations in the assumed nominal exponential atmosphere: any constant change in atmospheric density at sea level from the nominal value, constant changes in the decay factor of up to 10 percent, and changes in the density from the standard density up to 100 percent per 200,000 feet altitude.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., October 16, 1963.

APPENDIX

DESCRIPTION OF TERMINAL CONTROL TECHNIQUE

The control procedure, in its simplest form, can be described as a procedure by which perturbations of the vehicle trajectory are combined and used in specifying a control function which causes the vehicle trajectory to meet certain predetermined end conditions. A complete mathematical description of the terminal control technique can be found in a number of references (for example, ref. 5). Therefore, only a general discussion of the technique and the mechanics involved in adapting the procedure to the present study is included.

Adjoint Equations for Small Perturbations About a Nominal Trajectory

The equations describing small perturbations along a nominal trajectory are given by

$$\frac{d}{dt}(\delta x_i) = \sum_{j=1}^{n} \frac{\partial f_i}{\partial x_j} \delta x_j + \frac{\partial f_i}{\partial \phi} \delta \phi \tag{A1}$$

where $\frac{\partial f_i}{\partial x_j}$ and $\frac{\partial f_i}{\partial \phi}$ are evaluated along a nominal trajectory, so that they are functions of time, equation (A1) being linear with variable coefficients.

The set of equations adjoint to (A1) is defined as

$$\frac{dy_i}{dt} = -\sum_{j=1}^{n} \frac{\partial f_j}{\partial x_i} y_j \tag{A2}$$

where the negative transpose of the coefficient matrix $\frac{\partial f_i}{\partial x_j}$ occurs. The term $y_i(t)$ is the influence function associated with $x_i(t)$. This influence function shows how variations in the trajectory of the vehicle from a nominal trajectory will affect the value of certain trajectory variables at a later point along the trajectory. The influence function is obtained by integrating the adjoint equations "backwards" along the nominal trajectory. This backward integration is necessary since boundary conditions on the influence functions usually arise at the end of an entry.

In the present study the effects of variations from the nominal trajectory in altitude, flight-path angle, and velocity were considered. Thus equations (A2) are given by:

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \end{bmatrix} = - \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = - \begin{bmatrix} \frac{\partial \dot{h}}{\partial h} & \frac{\partial \dot{\gamma}}{\partial h} & \frac{\partial \dot{V}}{\partial h} \\ \frac{\partial \dot{h}}{\partial \gamma} & \frac{\partial \dot{\gamma}}{\partial \gamma} & \frac{\partial \dot{V}}{\partial \gamma} \\ \frac{\partial \dot{h}}{\partial V} & \frac{\partial \dot{\gamma}}{\partial V} & \frac{\partial \dot{V}}{\partial V} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \tag{A3}$$

The coefficients for equation (A3) are given in the following equations:

$$a_{11} = 0$$

$$a_{12} = \frac{V_R \cos \gamma_R}{57.3}$$

$$a_{13} = \sin \gamma_R$$

$$a_{21} = - \frac{57.3}{V_R m} BL \cos \phi_R$$

$$a_{22} = \frac{g \sin \gamma_R}{V_R} - \frac{V_R \sin \gamma_R}{r}$$

$$a_{23} = \frac{57.3}{V_R^2 m} L \cos \phi_R + 57.3 \frac{1}{r} + \frac{g}{V_R^2} \cos \gamma_R$$

$$a_{31} = \frac{D}{m} B$$

$$a_{32} = - \frac{g \cos \gamma_R}{57.3}$$

$$a_{33} = - \frac{2}{V_R m} D$$

Linear Prediction Technique

Since the final objective of each phase of the entry was different a somewhat different control scheme was used in guiding the vehicle during each phase. However, the basic principle involved in all cases was as follows:

The adjoint solutions and perturbations from a nominal trajectory were combined in such a manner that a predicted error in the final value of some trajectory variable from a desired value could be obtained. This error was then used to control the lift of the vehicle such that the predicted error would be reduced. For example, consider a hypothetical entry for which the vehicle is descending in the atmosphere from some initial altitude h_1 to some final altitude h_2. Suppose further that a scheme is desired which will predict the value of the velocity at h_2; this could be accomplished by using linear prediction techniques in the following manner.

A nominal trajectory is computed starting at h_1, with specified initial conditions in velocity and flight-path angle, and passing through h_2. This nominal trajectory should represent an "average" path through these altitudes. The adjoint differential equations are then integrated backwards along the trajectory from h_2 to h_1 with appropriate initial conditions.

These initial conditions are determined by considering the errors in final velocity due to each of the trajectory variables. An error $\delta V(t)$ at any time t along the trajectory would cause the same error $\delta V(T)$ at the final time T as $t \rightarrow T$. Hence, the initial value for the influence function $y_3(y_V)$ associated with velocity would be 1. The effect of errors $\delta\gamma(T)$ and $\delta h(T)$ on the final velocity would approach 0 as $t \rightarrow T$. Hence, the initial value of these influence functions $y_2(y_\gamma)$ and $y_1(y_h)$ would be zero.

For trajectories with small perturbations about the nominal trajectory the final velocity can now be predicted by the following equation which is equation (22) of reference 2 in the notation of the present report:

$$\delta_p V_T = \left(V_{R,T} - V_D\right) + y_1(t)\delta h + y_2(t)\delta\gamma + y_3(t)\delta V \qquad \text{(A4)}$$

and

$$V_T = V_D + \delta V_T$$

where V_D is the desired velocity at the final altitude h_2. A block diagram of this prediction system is given in figure 20. This velocity error could now be used in a control system to bring the final velocity closer to the desired final velocity. Various forms of this prediction equation (A4) were used in the present entry guidance system.

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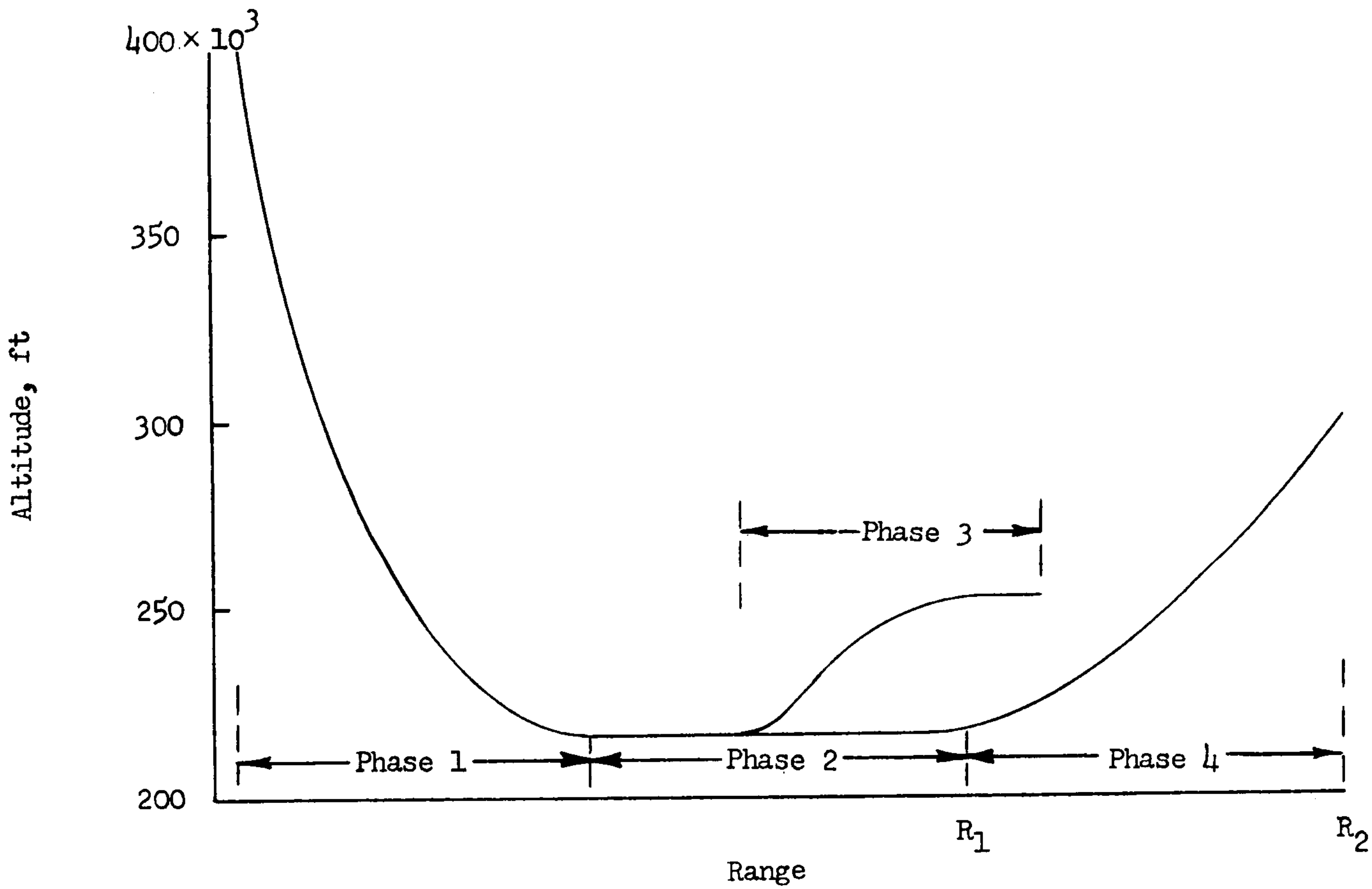


Figure 1.- Typical entry trajectory showing different guidance phases.

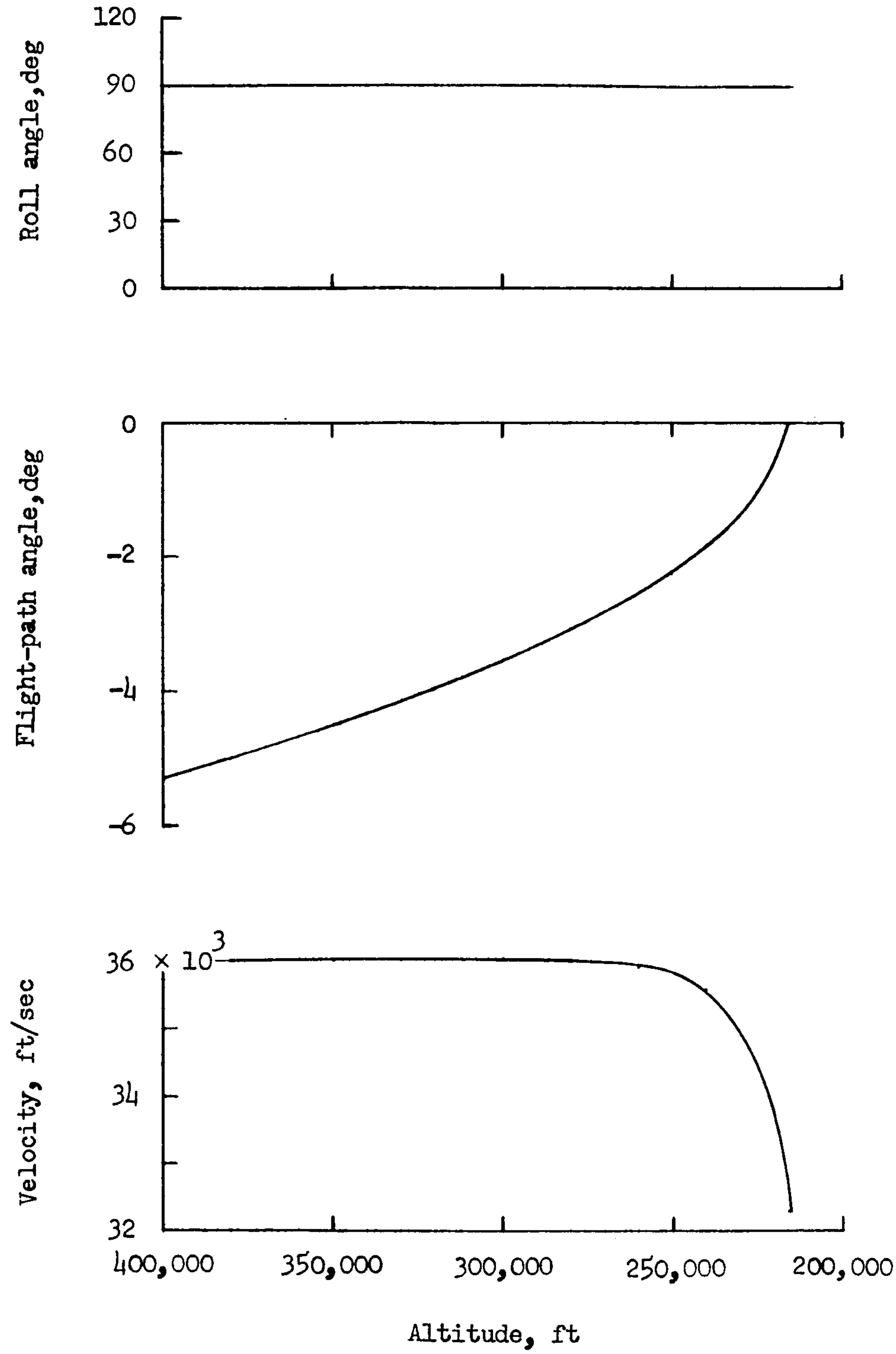


Figure 2.- Reference trajectories for phase 1. h_o = 400,000 feet; V_o = 36,000 ft/sec; $\gamma_o = -5.3^{\circ}$.

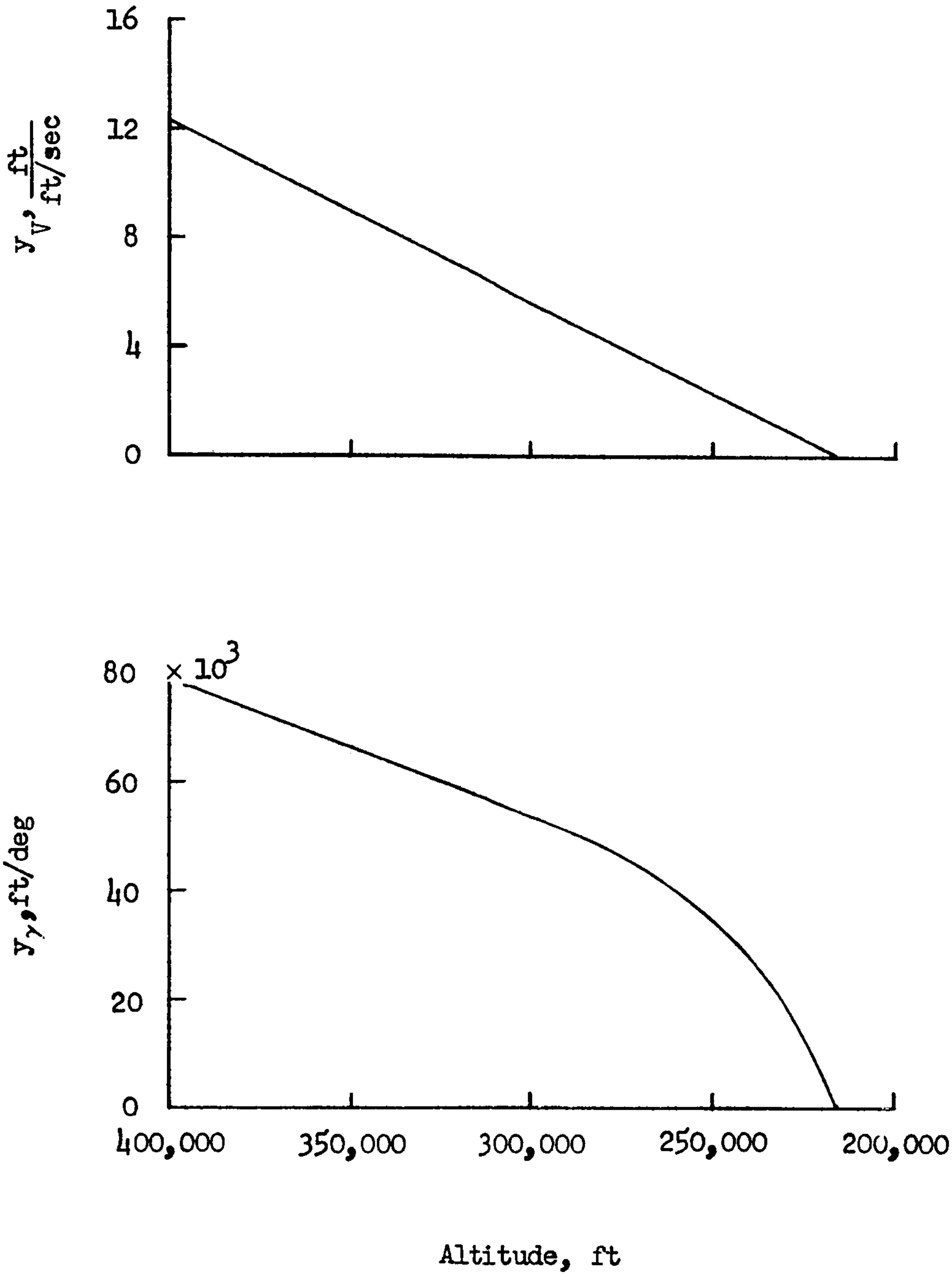


Figure 3.- Adjoint solutions for phase 1 reference trajectories.

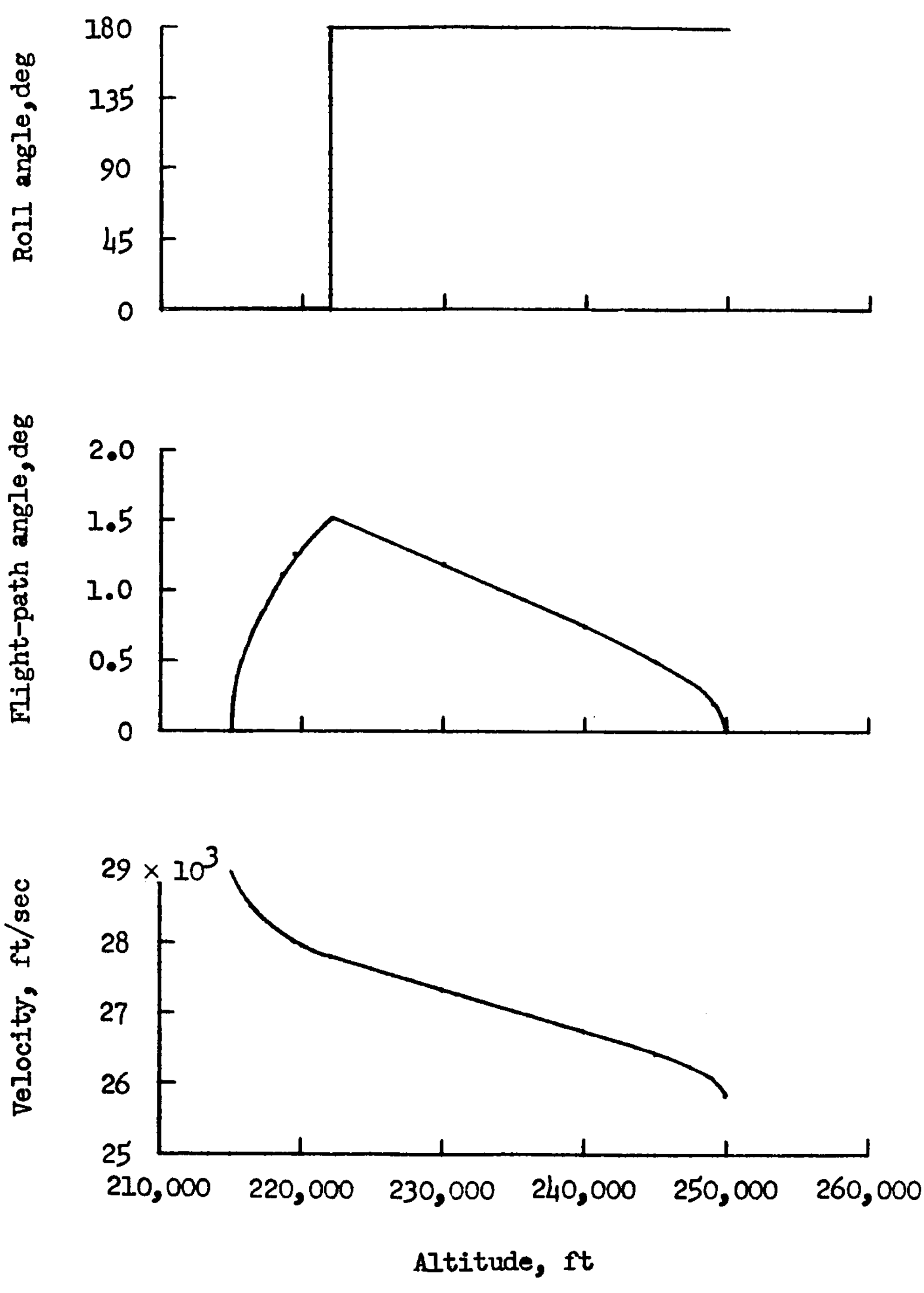


Figure 4.- Reference trajectories for phase 3. h_o = 215,500 feet; V_o = 29,000 ft/sec; $\gamma_o = 0^{\circ}$.

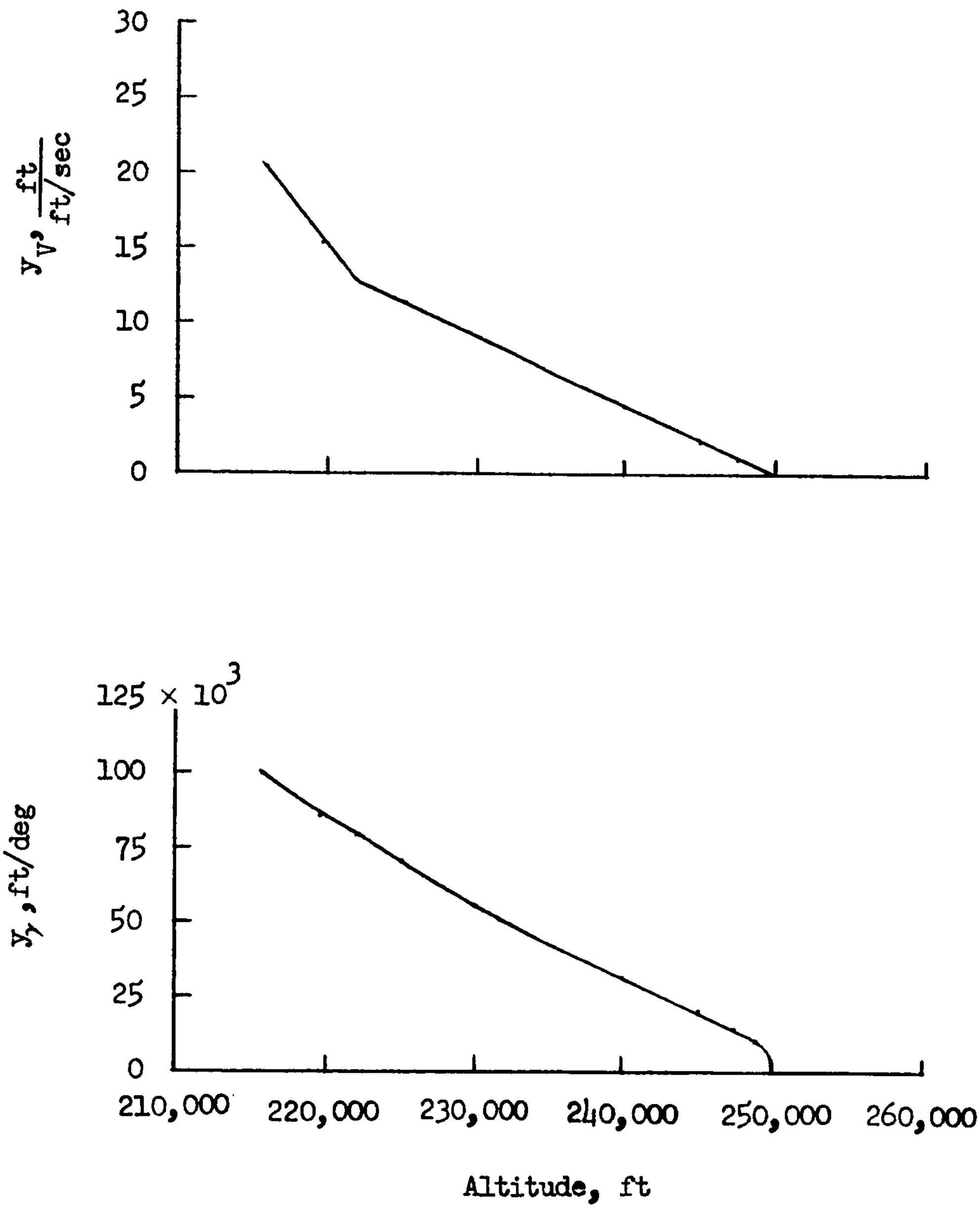


Figure 5.- Adjoint solutions for phase 3 reference trajectories.

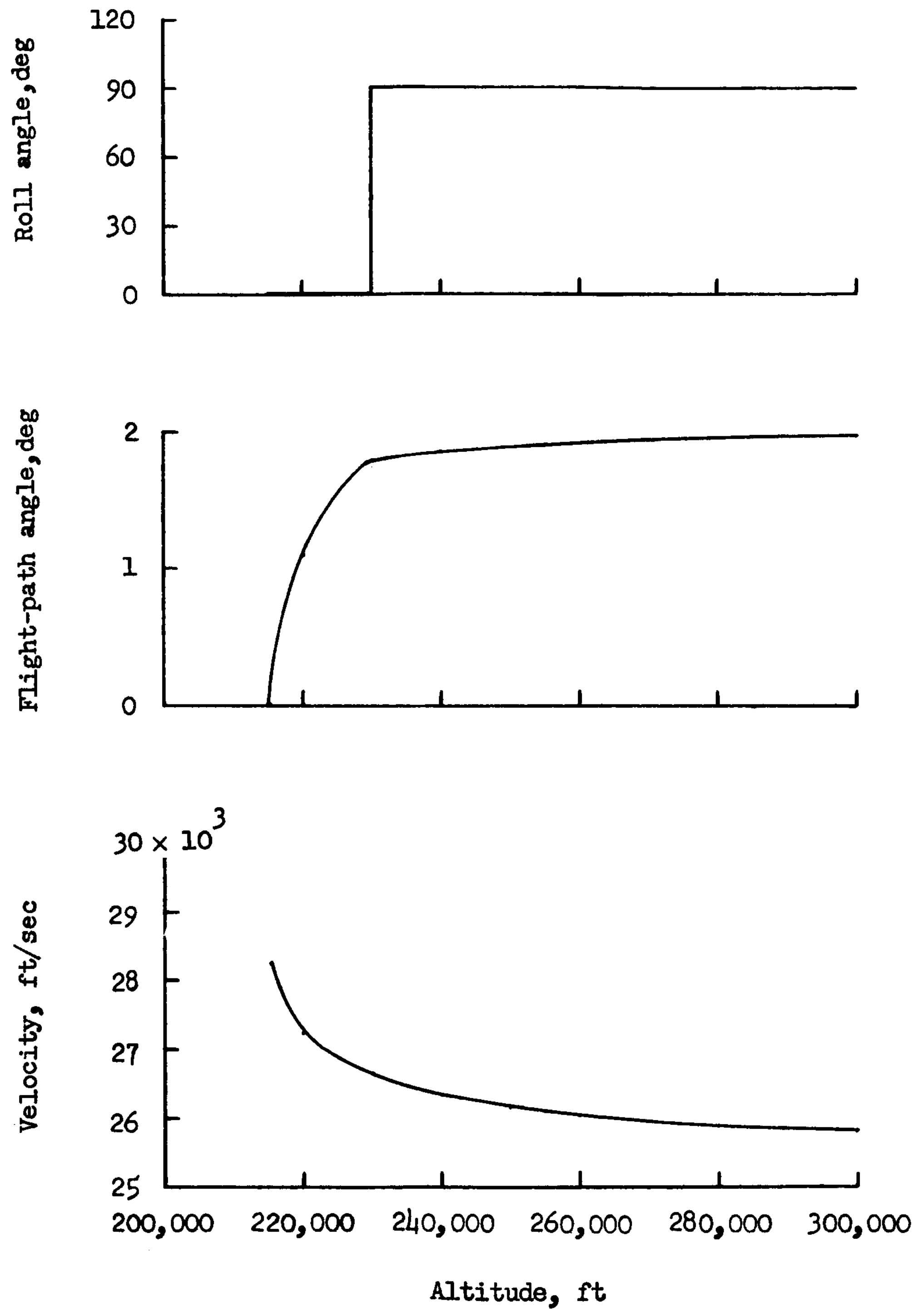
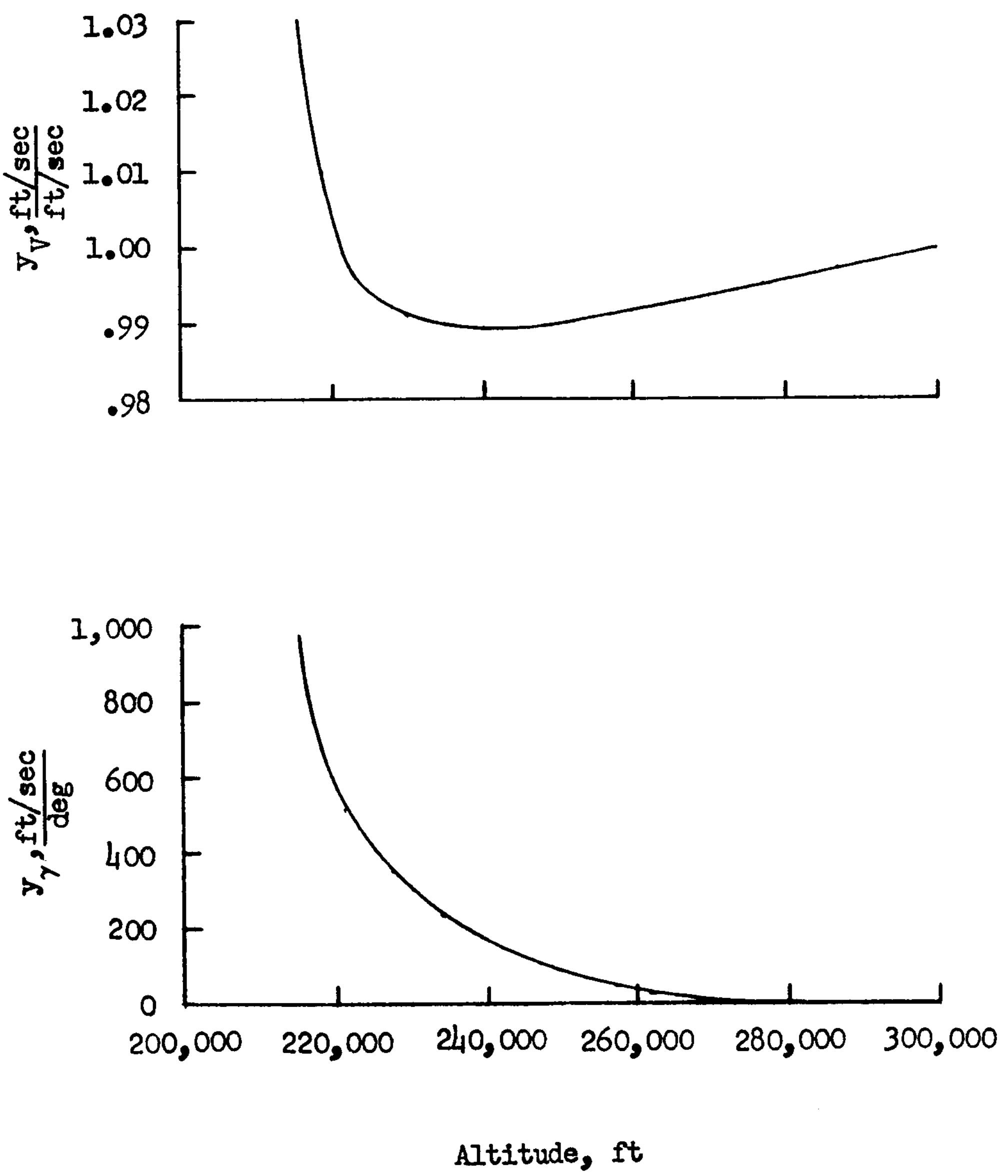
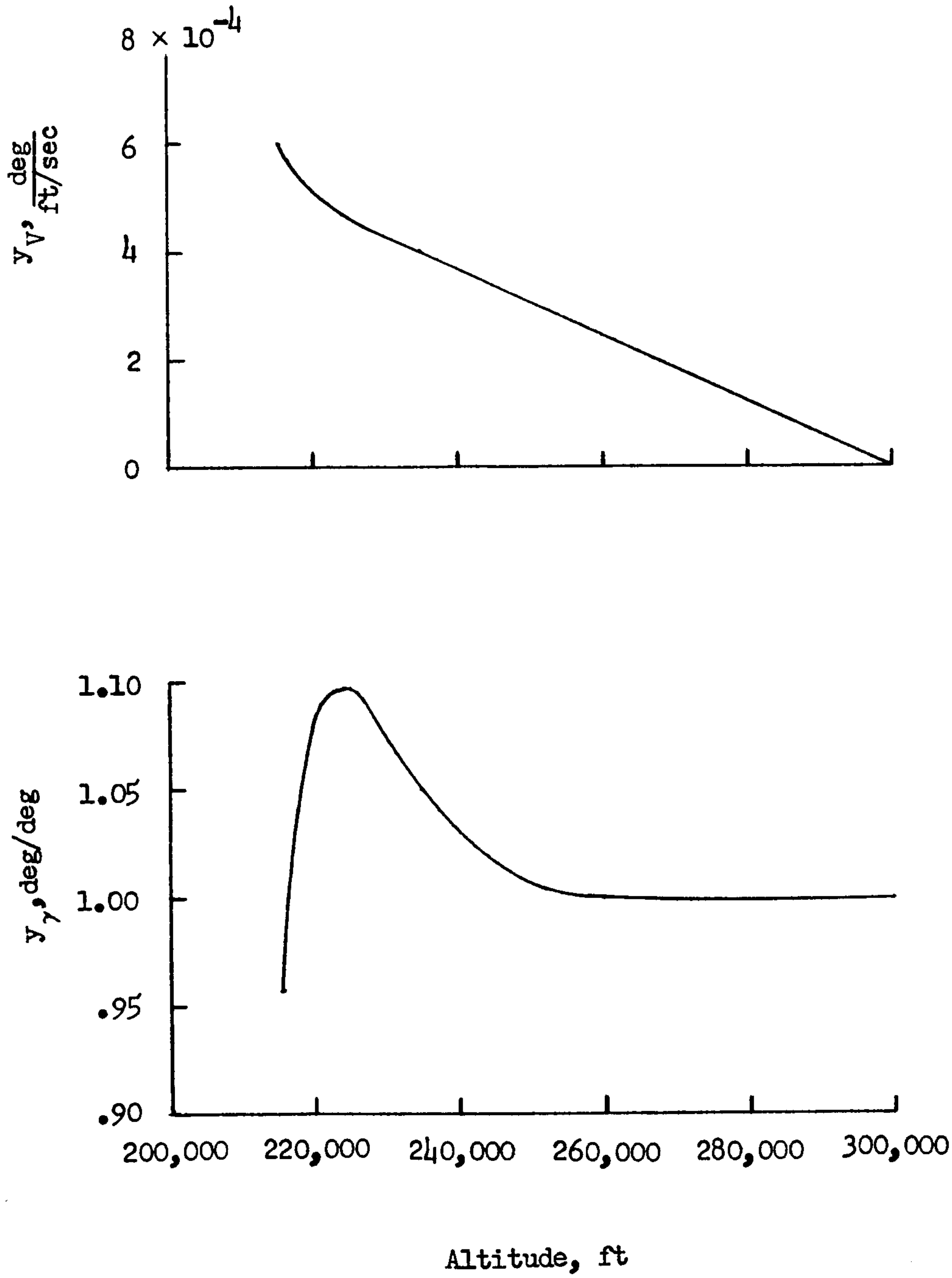


Figure 6.- Reference trajectories for phase 4. h_o = 215,500 feet; V_o = 28,250 ft/sec; $\gamma_o = 0^\circ$.



(a) Influence functions for velocity prediction.

Figure 7.- Adjoint solutions for phase 4 reference trajectories.



(b) Influence functions for flight-path-angle prediction.

Figure 7.- Concluded.

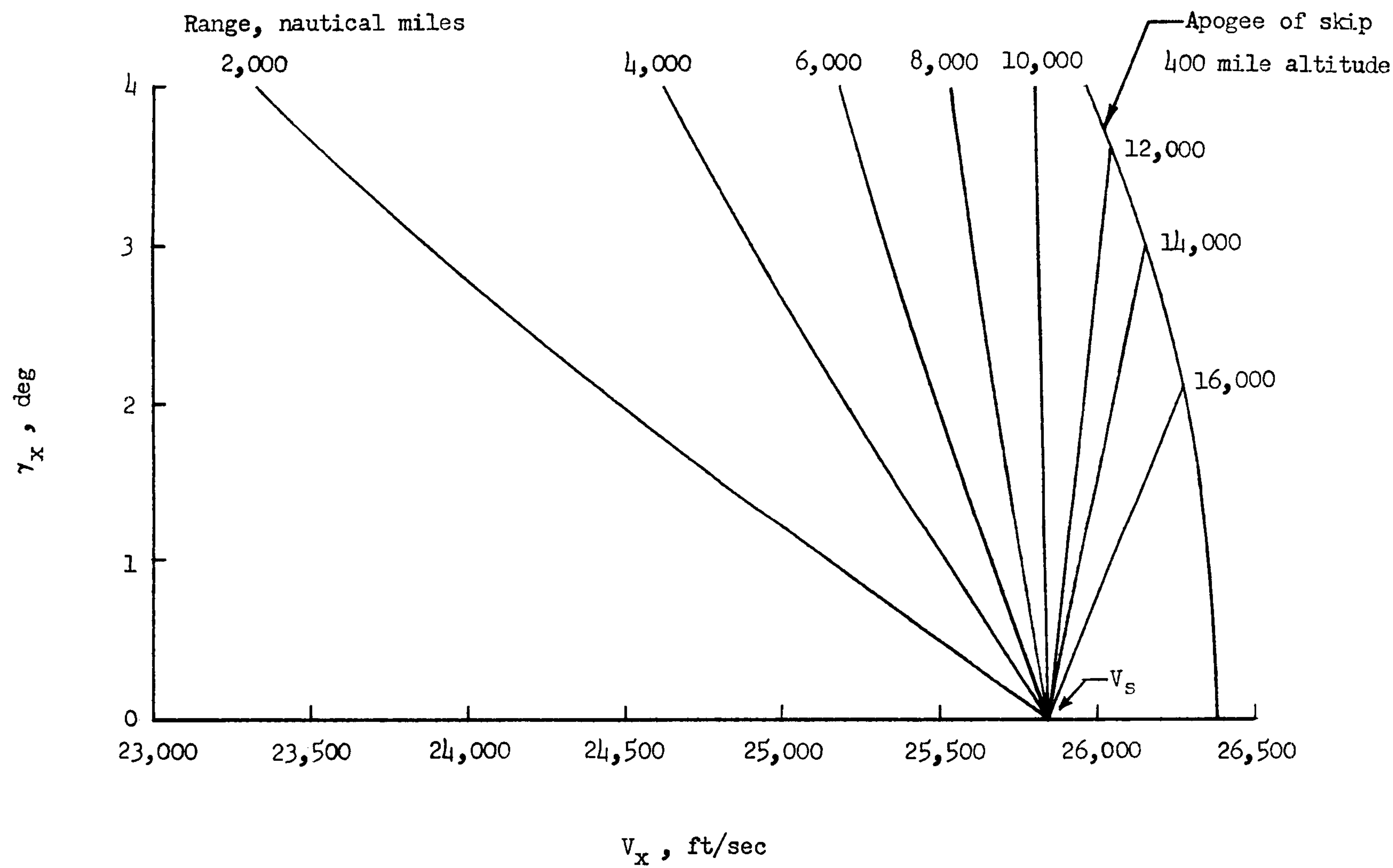


Figure 8.- Longitudinal range outside earth atmosphere (from 300,000 feet to 400 miles altitude) as a function of exit conditions. (From ref. 6.)

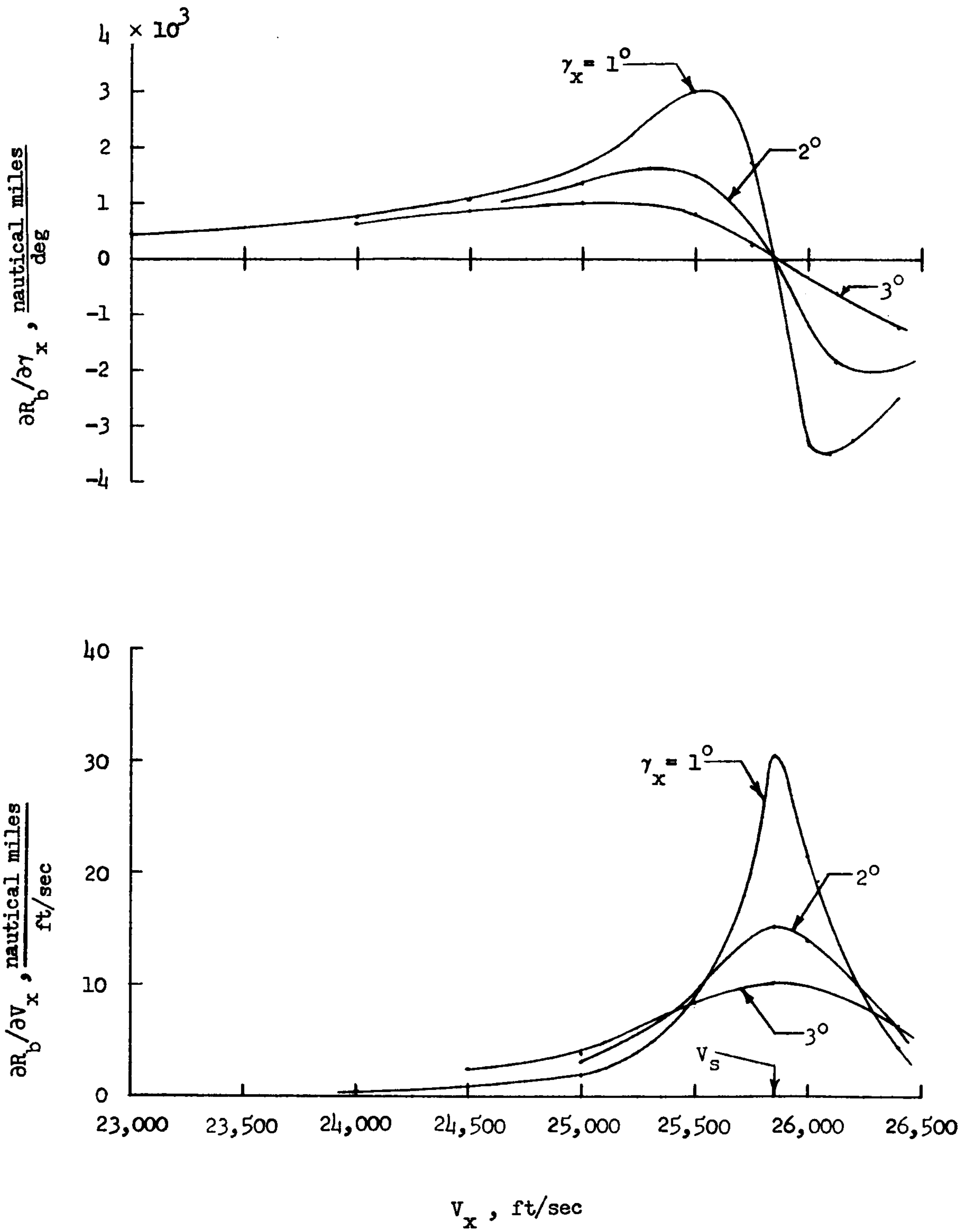


Figure 9.- Dependence of range outside the atmosphere to changes in exit flight-path angle or exit velocity. (From ref. 6.)

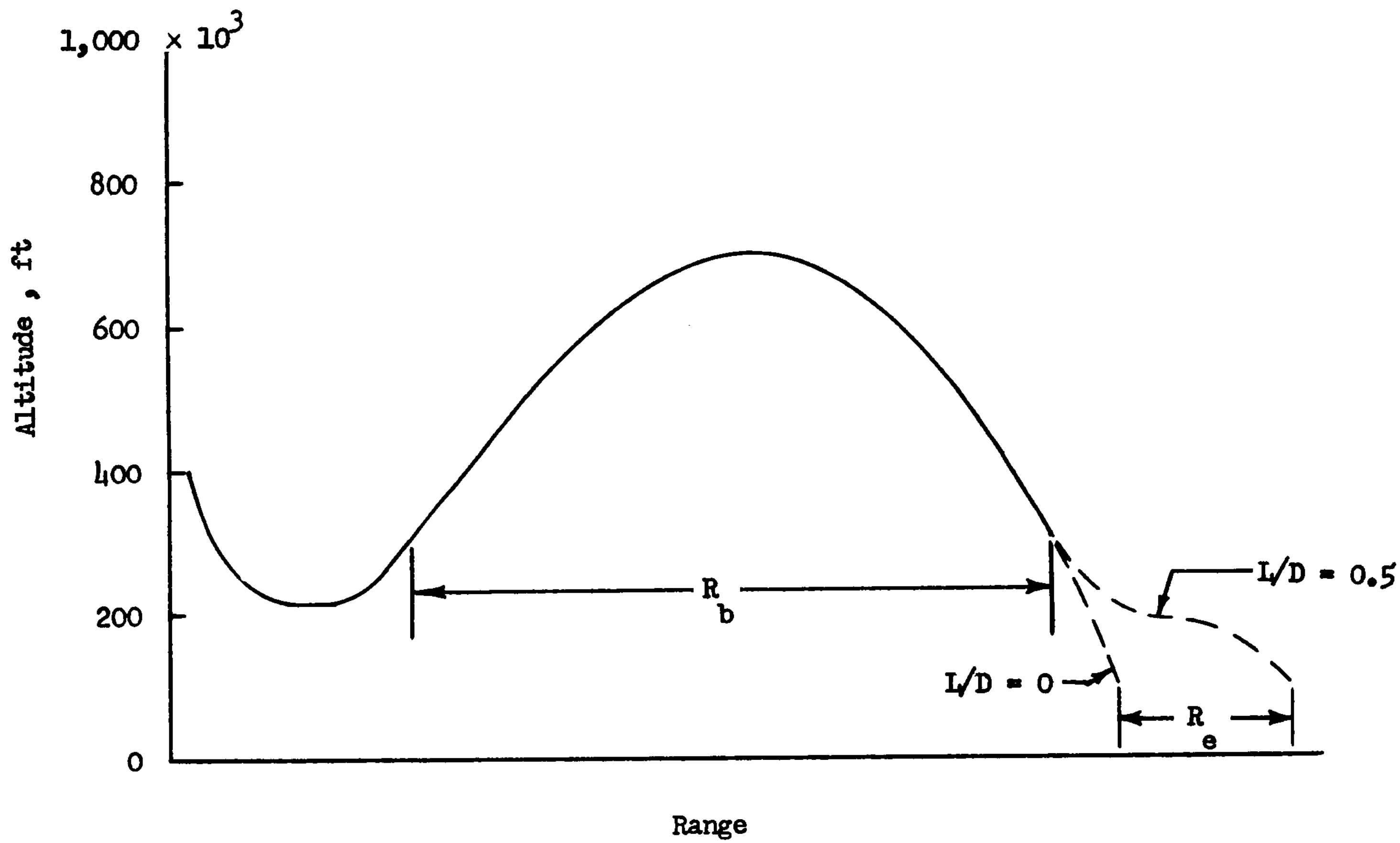


Figure 10.- Range variation for a typical skip entry.

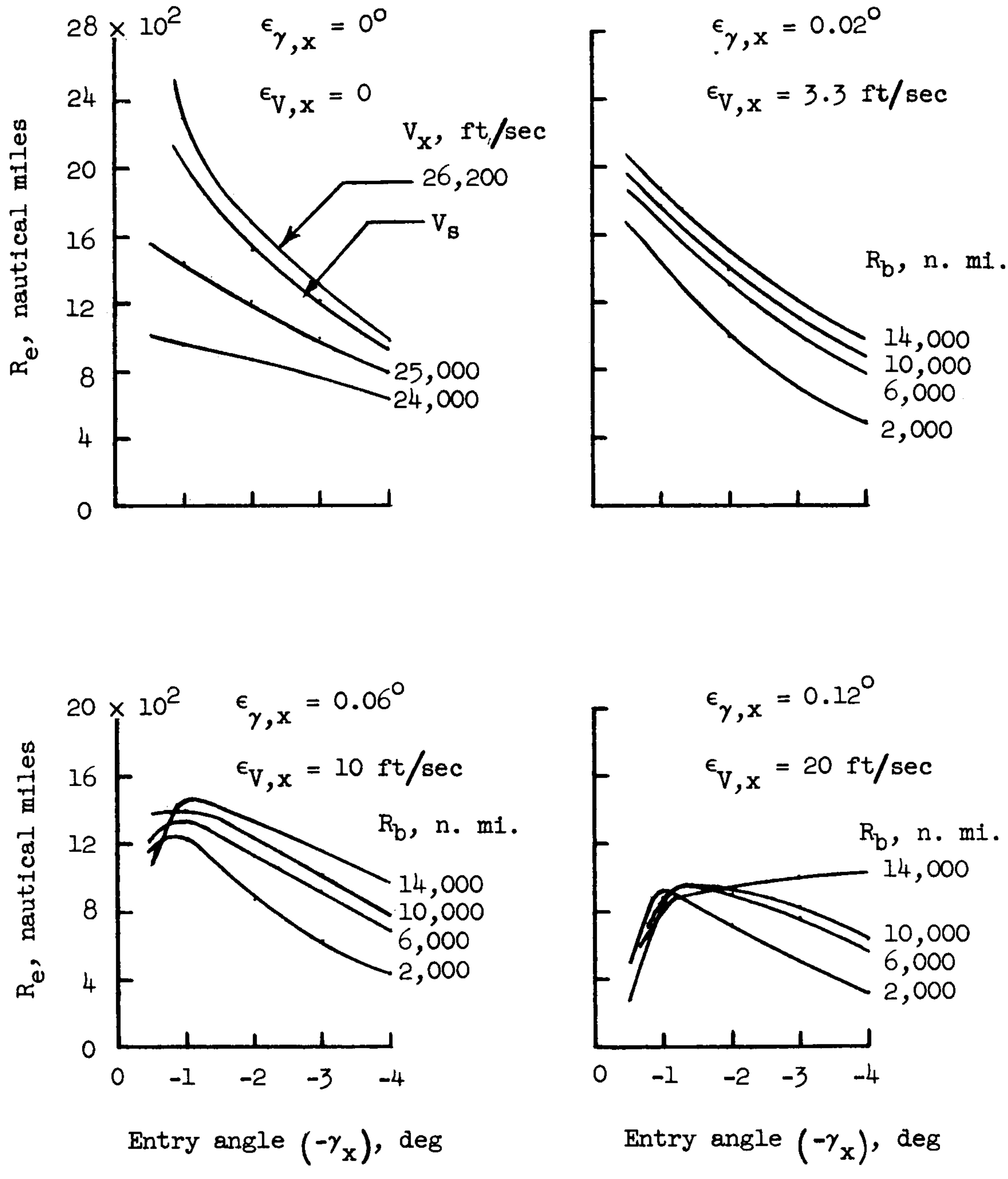


Figure 11.- Variation in range capability during reentry with entry angle and ballistic range for different assumed errors in exit velocity and flight-path angle.

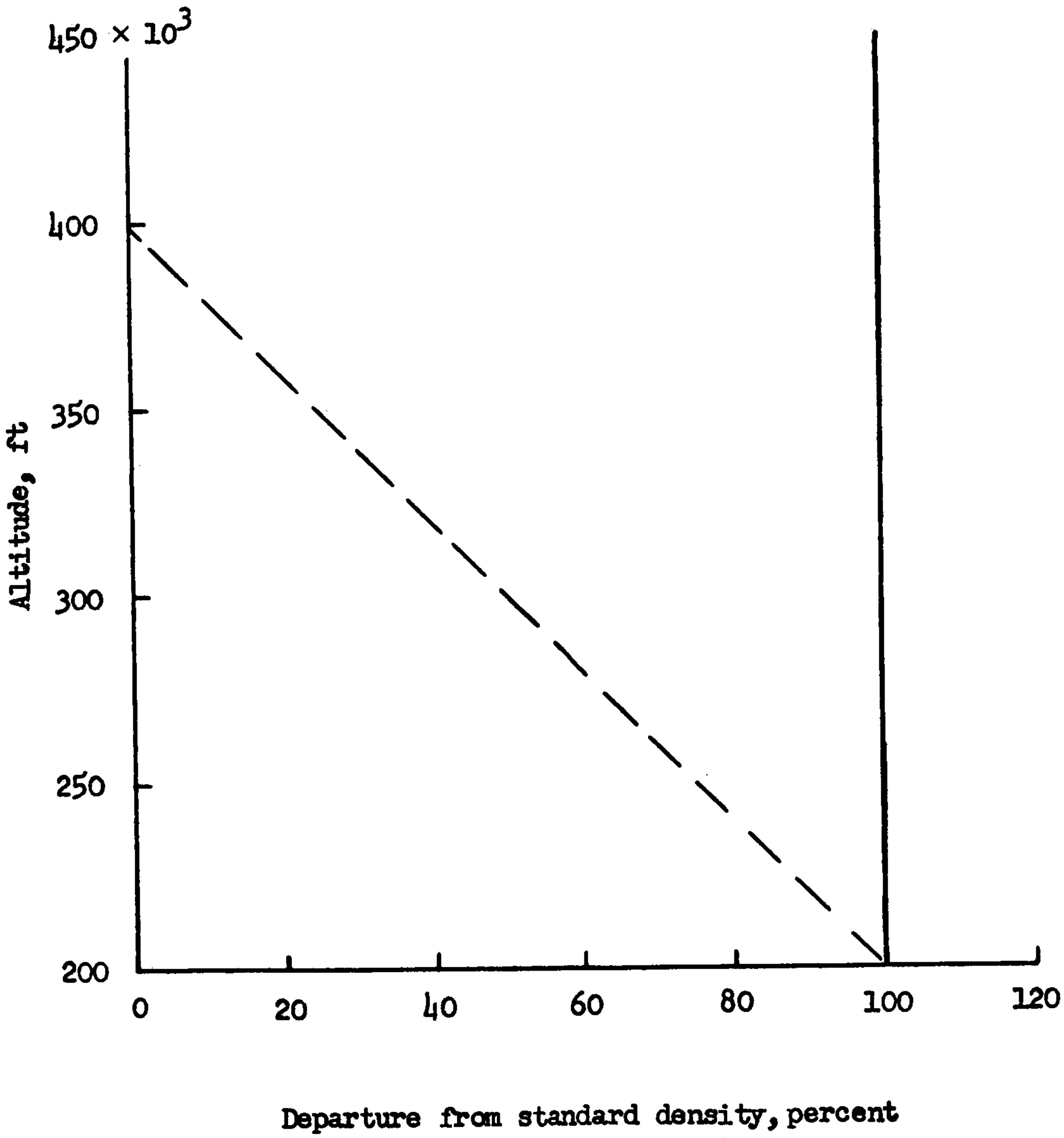


Figure 12.- Possible variations of the atmospheric density from the standard density.

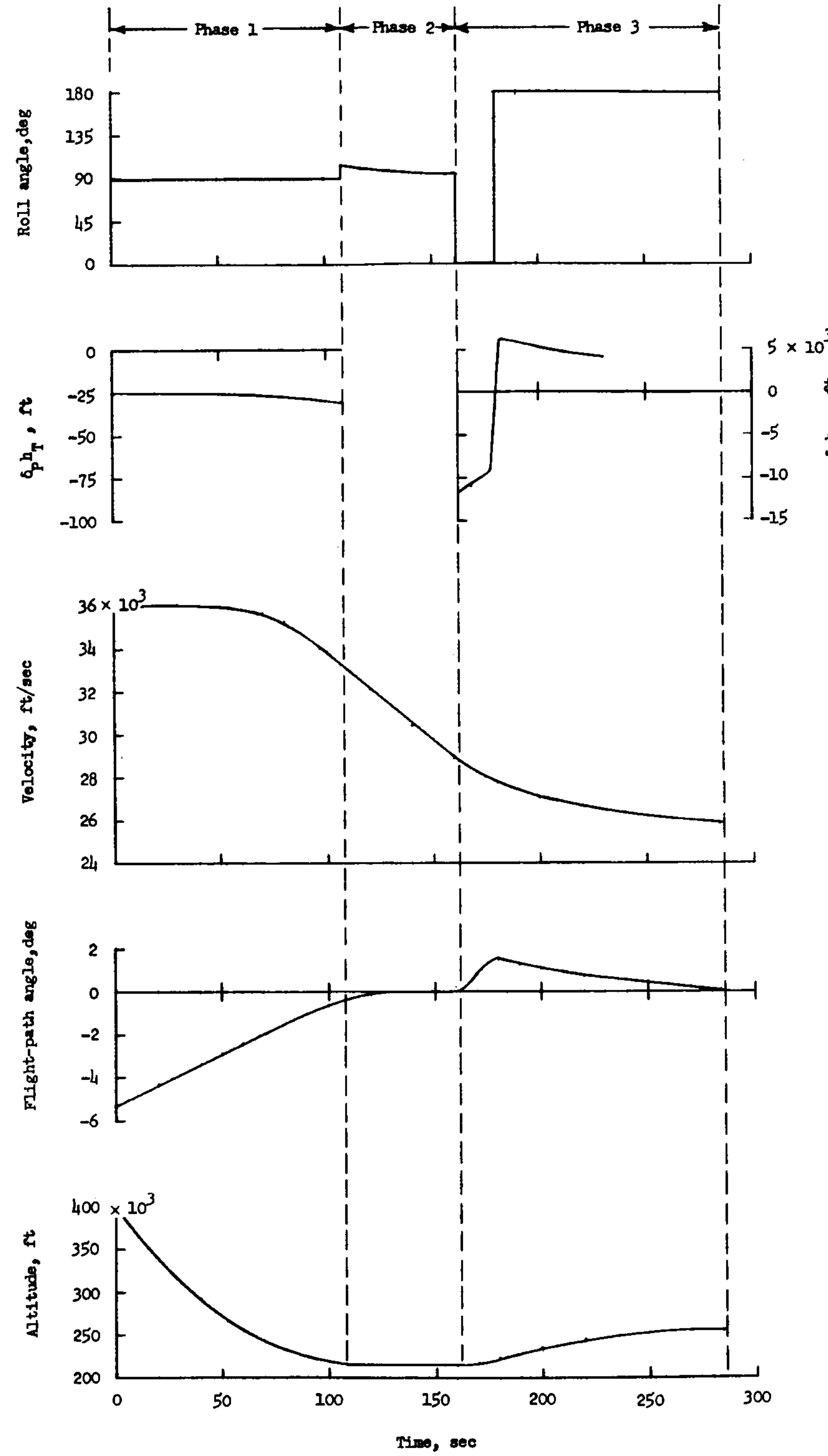


Figure 13.- Time histories for a direct entry with a desired range of less than 4,200 nautical miles. h_o = 400,000 feet; V_o = 36,000 ft/sec; γ_o = -5.3°.

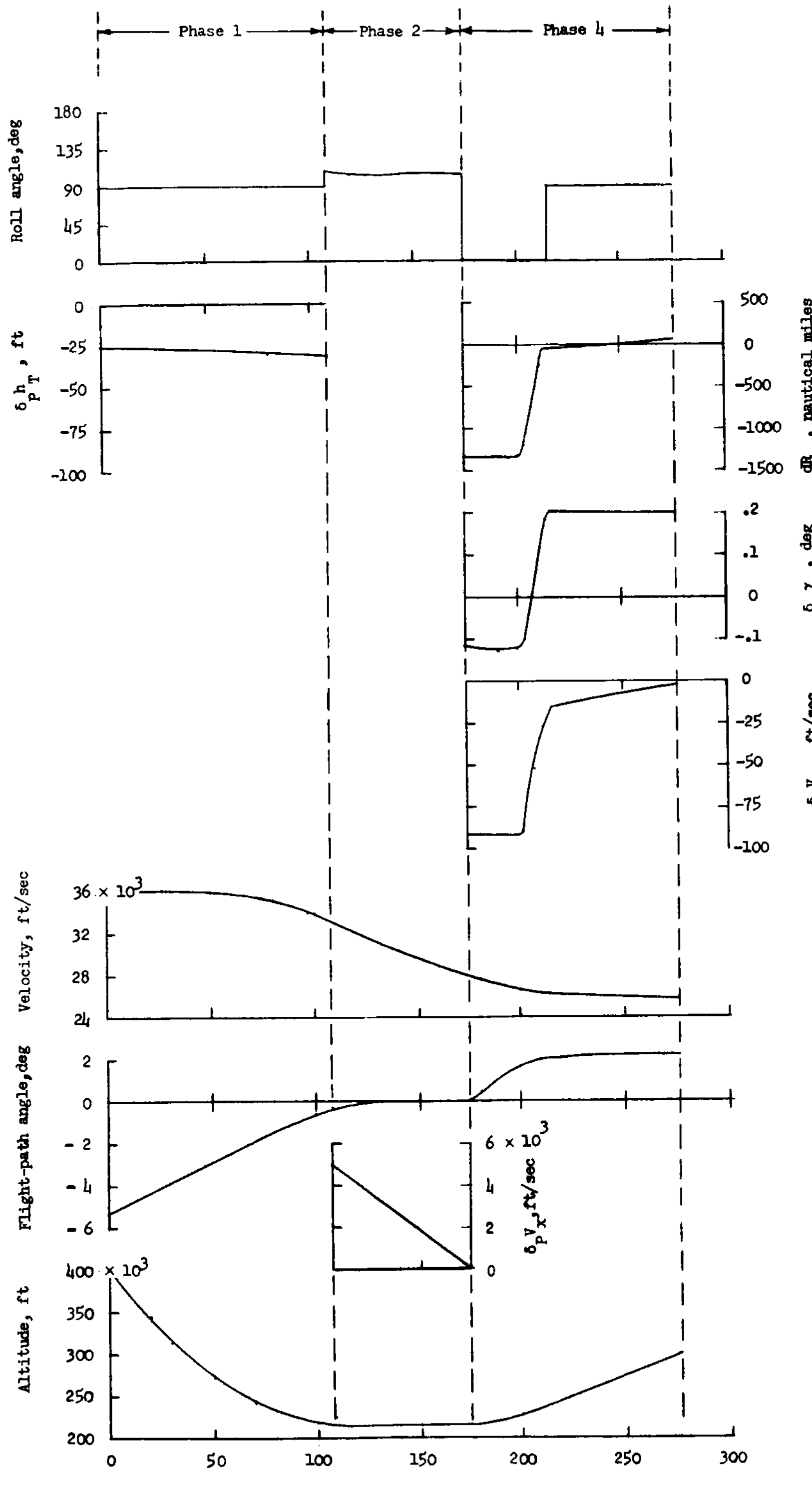


Figure 14.- Time histories for skip entry with an exit velocity less than satellite velocity. h_o = 400,000 feet; V_o = 36,000 ft/sec; γ_o = -5.3°; R_D = 10,000 nautical miles.

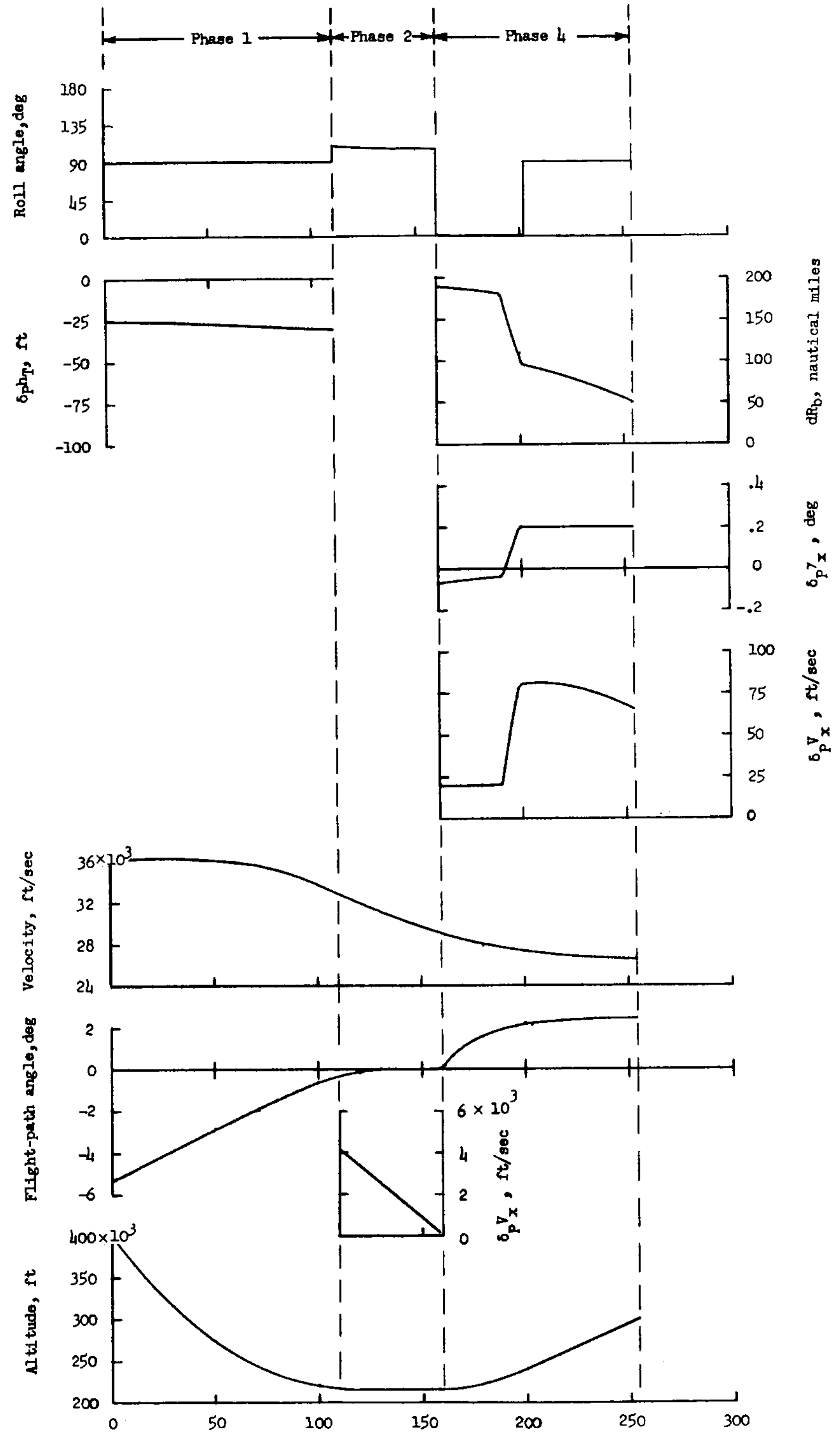


Figure 15.- Time histories for skip entry with an exit velocity greater than satellite velocity. h_o = 400,000 feet; V_o = 36,000 ft/sec; γ_o = -5.3°; R_D = 16,500 nautical miles.

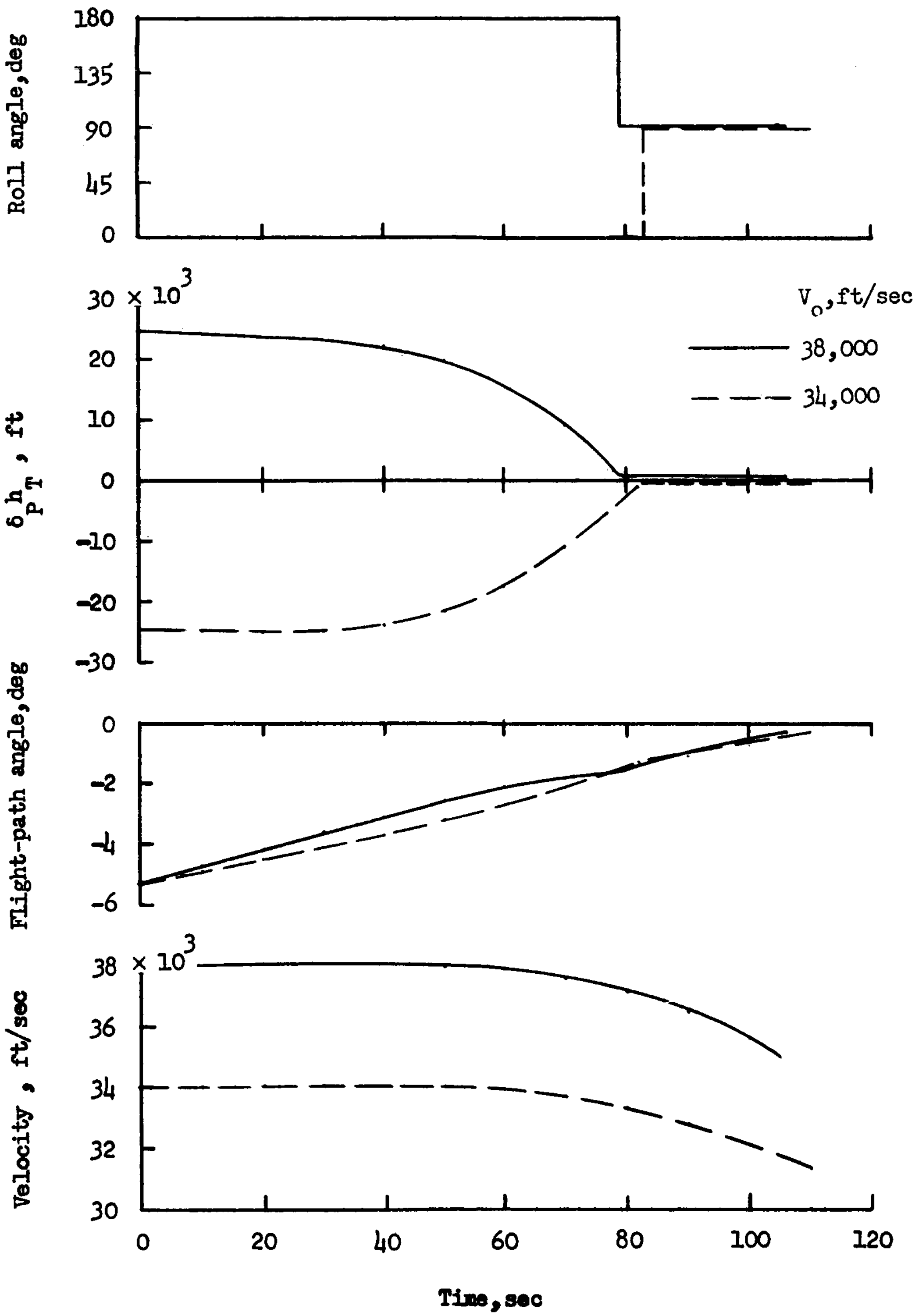


Figure 16.- Time histories of phase 1 of entries with off-nominal conditions for the initial entry velocity. h_o = 400,000 feet; γ_o = -5.3°.

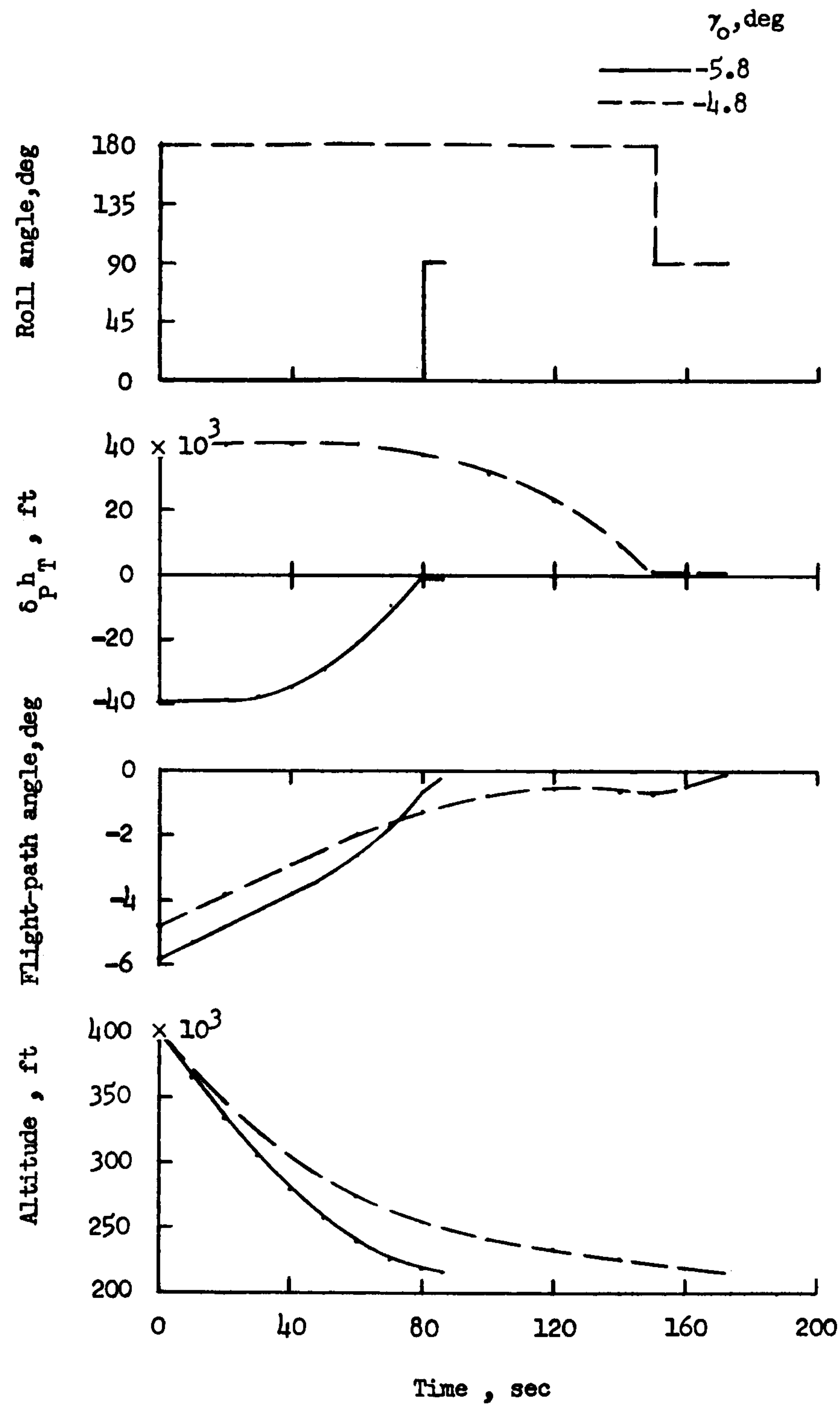


Figure 17.- Time histories of phase 1 of entries with off-nominal conditions for the initial flight-path angle. h_o = 400,000 feet; V_o = 36,000 ft/sec.

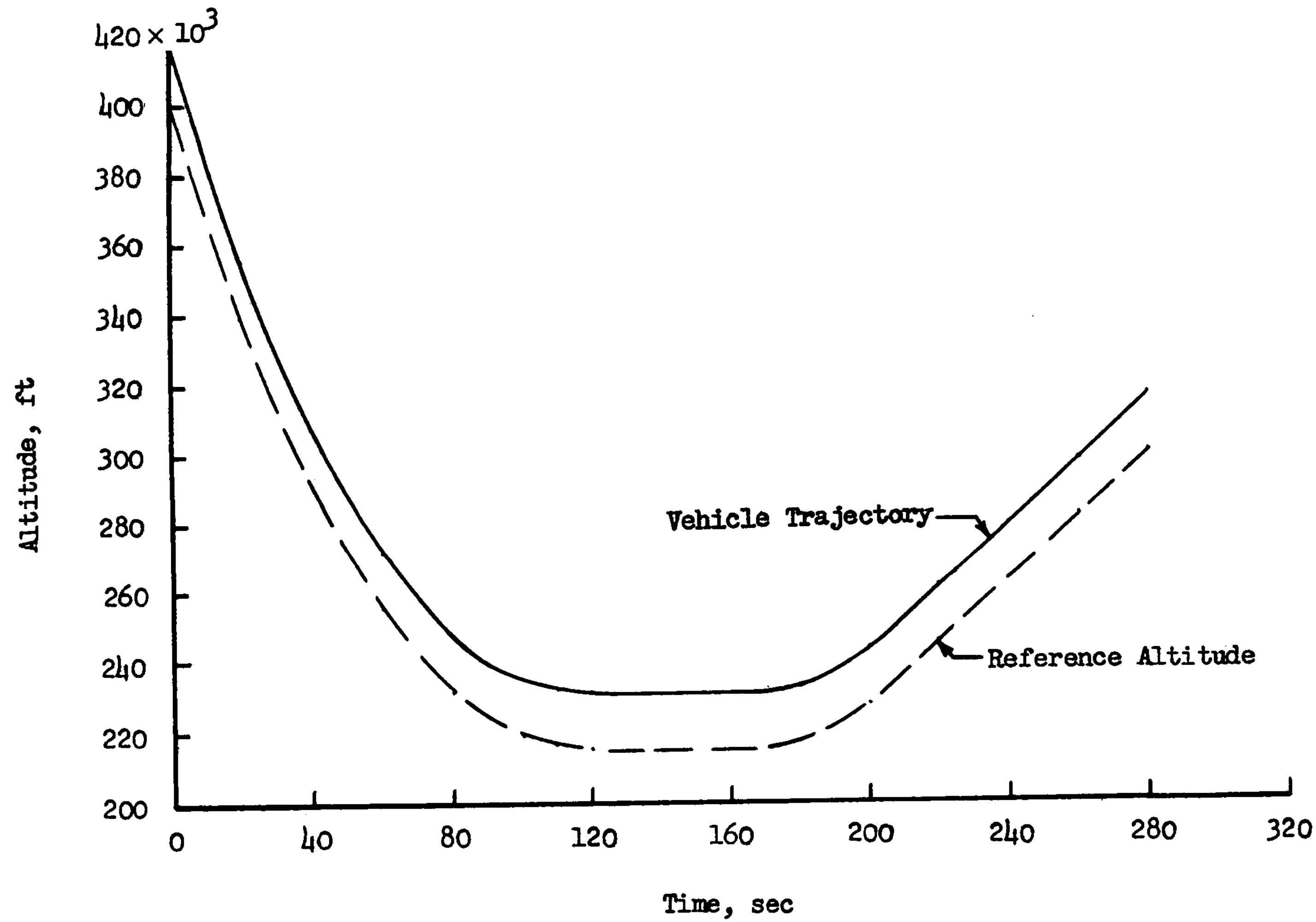


Figure 18.- Time history of altitude for a skip entry with $\rho_c = 2\rho_{c,R}$. h_o = 400,000 feet; V_o = 36,000 ft/sec; $\gamma_o = -5.3^o$.

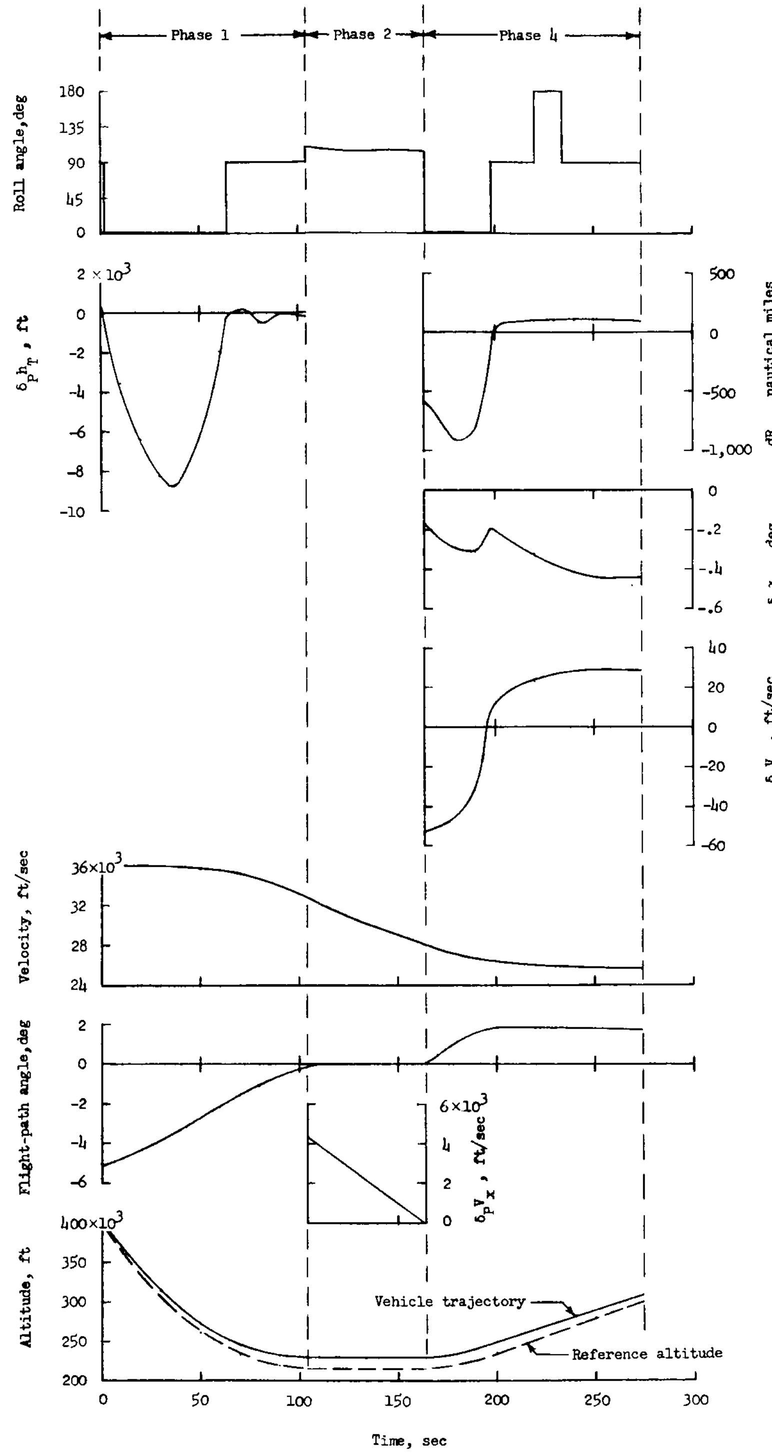


Figure 19.- Time histories of a skip entry for which the standard atmospheric density was altered linearly as a function of altitude. h_o = 400,000 feet; V_o = 36,000 ft/sec; $\gamma_o = -5.3^o$.

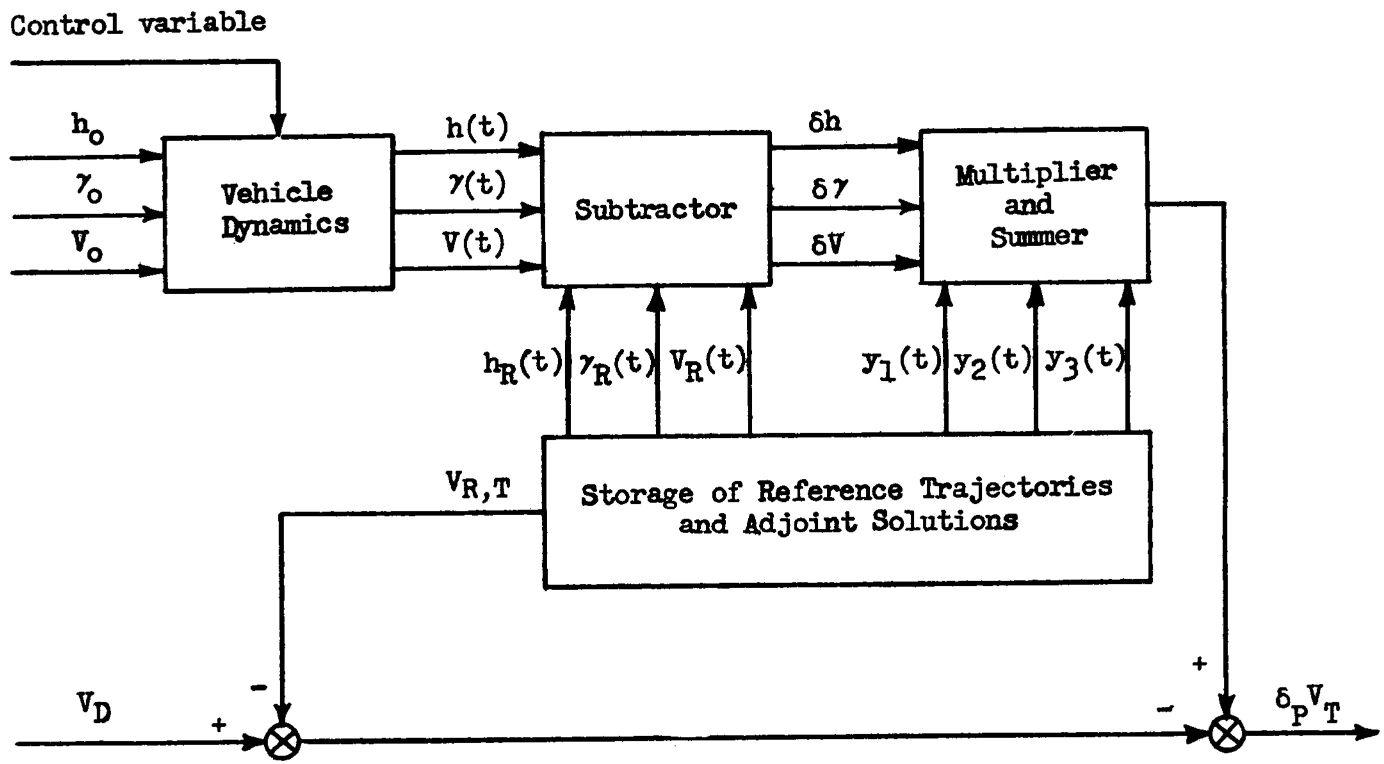


Figure 20.- Block diagram illustrating the use of terminal controller techniques in predicting the terminal value for some trajectory variable.